

THEORIA LUNÆ

A T A R E

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AUCTORE

TOBIAMAYER.

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THEORIA LUNE

EXTRA

SYSTEMA NEWTONIANUM

AUCTORE

TOMIA MARY R.

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P R Æ F A T I O.

MODUM, quo inæqualitates in motu Lunæ ex theoria investigavi, non ideo hic exhibeo, ut inde demonstrem fidem ac veritatem tabularum mearum lunarium; habet enim theoria hoc incommodi, ut plures inde inæqualitates accurate deduci nequeant, nisi quis forte calculum hunc, in quo jam omnem fere patientiam meam exhausti, longe adhuc curatius persequatur; sed ut hoc saltem ostendam, nullum ex theoria argumentum contra bonitatem tabularum mearum peti posse. Hoc vero evidentissime colligitur inde, quia inæqualitates quæ in tabulis correctis et ad plurimarum observationum nutum castigatis reperiuntur, ab his, quæ ex theoria sola prodeunt, nullibi fere plus dimidio minuto discrepant. Adeo ut vix major consensus in re tot tantisque difficultatibus undique septa desiderari possit, satisque adpareat, exiguos hosce errores potius à parte calculi theoretici, quam tabularum, proficisci. Idipsum vero extra dubitationem omnem collocabitur, si aliorum, imprimis celeb. EULERI, CLAIRAUTI,

ac

P R Æ F A T I O.

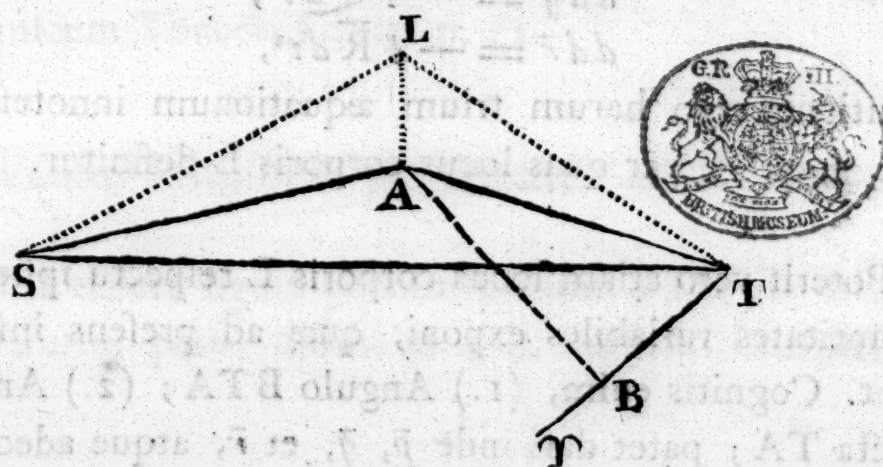
ac D'ALEMBERTI, ex hac eadem theoria deducti numeri cum meis, five tabulis, five calculis theoreticis comparentur; quippe à quibus nonnunquam tribus pluribusve adeo minutis differunt, neque magis inter se ipsos, nisi paucissimis in locis, consentiunt. Hoc igitur potissimum isto meo calculo à me demum certissime evictum puto, quod dubium potius reddiderunt quam confirmarunt virorum modo laudatorum conatus, nempe egregiam attractionis NEWTONIANÆ theoriam, etiam in his minimis cum observationibus congruere.



T H E O R I A

THEORIA LUNÆ.

Investigatio in motum corporis L ad T relatum, sollicitati
à viribus quibuscunque.



§ 1. **C**ONCIPIATUR planum STA immobile transiens per
T, inque eo recta TT positionis constantis; ex loco
corporis L demittatur in hoc planum normalis LA,
jungaturque recta AT, ex A autem ad TT ducatur normalis AB.
Quodsi ergo ad quodvis tempus adsignari possit valor rectarum TB,
AB, AL, cognoscetur etiam locus corporis L, qui quæritur.

§ 2. Sit igitur $BT = \bar{p}$, $AB = \bar{q}$, $LA = \bar{r}$, et cum vires omnes,
quæ motum corporis L respectu ipsius T afficiunt, ad tres directiones
secundum BT, AB, et LA reduci possunt, ponatur vis secundum
directionem ipsi BT parallelam $= \bar{P}$; vis secundum directionem
B parallelam


$$\begin{aligned} dd\bar{p} &= -\frac{1}{2}\bar{P}dt^2, \\ dd\bar{q} &= -\frac{1}{2}\bar{Q}dt^2, \\ dd\bar{r} &= -\frac{1}{2}\bar{R}dt^2, \end{aligned}$$

$$d d \bar{r} = -\frac{1}{3} \bar{R} d t^2,$$

ex resolutione ergo harum trium æquationum innotescunt valores
rectarum p, q, r , per quas locus corporis L definitur.

$$\begin{aligned} \text{AT} &= av, \text{ erit} \\ \text{BT} &= \bar{p} = av \cos. \varphi, \\ \text{AB} &= \bar{q} = av \sin. \varphi, \\ \text{LA} &= \bar{r} = av \tan g. l, \end{aligned}$$

$$L A = \vec{r} = a v \text{ tang. } l,$$

hinc autem substitutione in superioribus æquationibus rite facta, inveniuntur tres aliæ, in quibus $\bar{p}$, $\bar{q}$, et $\bar{r}$, haud amplius insunt, et ex quarum resolutione motus corporis pariter innotescet; erit autem

$$\text{III. } dd(v \text{ tang. } l) = -\frac{1}{2} \frac{dr^2}{d} \bar{R}.$$

§ 4. Quoniam vero $\bar{P} \cos. \varphi + \bar{Q} \sin. \varphi$ exprimit vim compo-
sitam ex $\bar{P}$ et $\bar{Q}$ ortam et agentem secundum directionem ipsi AT
parallelam, $\bar{Q} \cos. \varphi - \bar{P} \sin. \varphi$ vero vim reliquam ipsi AT nor-
malem et plano STA parallelam, sit ista vis $\bar{P} \cos. \varphi + \bar{Q} \sin. \varphi = S$,
hæc vero $\bar{Q} \cos. \varphi - \bar{P} \sin. \varphi = T$, erit

$$\text{I. } d d v - v d \varphi^2 = -\frac{d r^2}{a} S,$$

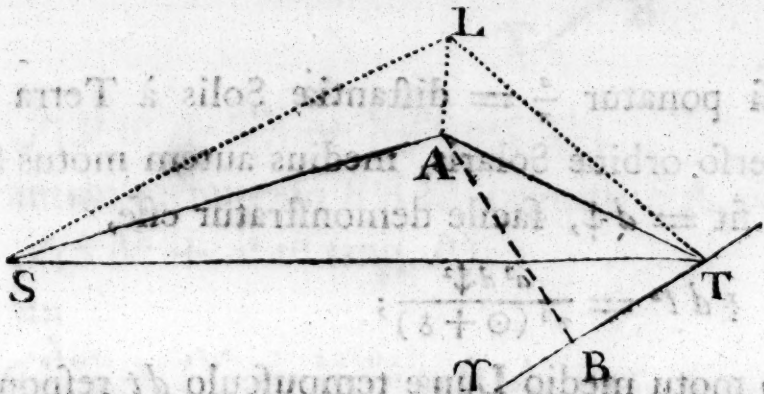
$$\text{II. } 2 d v d \varphi + v d d \varphi = -\frac{d r^2}{a} T,$$

$$\text{III. } d d (v \text{ tang. } l) = -\frac{d r^2}{a} \bar{R},$$

quæ æquationes istæ fere sunt, ad quas primus hoc problema reduxit
Cel. Eulerus, in Differtatione de Motibus Jovis et Saturni, qua præ-
mium meruit ab Academia Scient. Paris. ad an. 1748. propositum.
Vide etiam ejusdem Theoria Lunæ, p. 12.

Adapplicatio solutionis præcedentis ad Lunam.

§ 5. Si in hac eadem figura ponatur planum STA esse ecliptica *,
L Luna, T Terra, patet, formulas § 4. inventas comprehendere



motum Lunæ, foreque φ longitudinem ejus veram à puncto cœli
quovis immobili T numeratam, l latitudinem, et av distantiam à
Terra curtatam AT five projectionem radii vectoris LT.

* Et si eclipticæ planum ab attractione tam Lunæ quam aliorum planetarum à situ suo paulum
dimoveatur, licet tamen in presenti negotio tam immobilem haberi, ob eandem rationem, qua
plures alias quantitates minimas in hoc calculo omittere cogimur.

§ 6. Junctis autem SL , LT , ST , ex positis

Distantia Solis à Terra $ST = \frac{a}{\pi}$, massa Solis $= \odot$,

Distantia Solis à Luna $SL = \frac{a}{\pi}$, massa Terræ $= \delta$,

Longitudine vera Solis $\gamma TS = s$, massa Lunæ $= \nu$,

erit angulus STA , five distantia Lunæ à Sole $= \phi - s$, quem ponamus $= \omega$; vires autem, quibus Lunæ motus respectu Terræ, quam immotam concipi oportet, afficitur, erunt juxta theoriam Newtoni istæ,

$$\text{Vis secundum } LT = \frac{(\nu + \delta) \cos. l^3}{a^2 \omega^2},$$

$$\text{Vis secundum } LS = \frac{\odot \pi^2}{a^2 \nu^2},$$

$$\text{Vis secundum } ST = \frac{\odot \pi^2}{a^2 s^2},$$

unde resolutione rite instituta oriuntur vires

$$S = \frac{(\nu + \delta) \cos. l^3}{a^2 \omega^2} + \frac{\odot \pi^2 \nu}{a^2 \nu^3} + \left(\frac{\odot \pi^2}{a^2 s^2} - \frac{\odot \pi^2 \nu}{a^2 \nu^3} \right) \cos. \omega,$$

$$T = - \left(\frac{\odot \pi^2}{a^2 s^2} - \frac{\odot \pi^2 \nu}{a^2 \nu^3} \right) \sin. \omega,$$

$$\bar{R} = \frac{(\nu + \delta) \tan. l \cos. l^3}{a^2 \omega^2} + \frac{\odot \pi^2 \nu \tan. l}{a^2 \nu^3}.$$

§ 7. Jam si ponatur $\frac{a}{\pi} =$ distantia Solis à Terra mediæ, five semiaxi transverso orbitæ Solaris, medius autem motus Solis tempori dt respondens sit $= d\downarrow$, facile demonstratur esse,

$$\frac{1}{2} d t^2 = \frac{a^3 d \downarrow^2}{\pi^3 (\odot + \delta)};$$

hoc est, posito motu medio Lunæ tempusculo dt respondente $= dq$, ejusque ratione ad motum medium Solis, ut $1 : \bar{n}$,

$$\frac{1}{2} d t^2 = \frac{a^3 \bar{n}^2 dq^2}{\pi^3 (\odot + \delta)}.$$

§ 8. Substituantur valores hic inventi pro S , T , $\bar{R}$, et $\frac{1}{2} d t^2$ in æquationibus, § 4. exhibitis, ponaturque brevitatis causa

$$\frac{\bar{n}(\odot + \delta)}{(\odot + \delta)\pi^3} = g,$$

$$\frac{\odot \bar{n}^2}{\odot + \delta} = n^2,$$

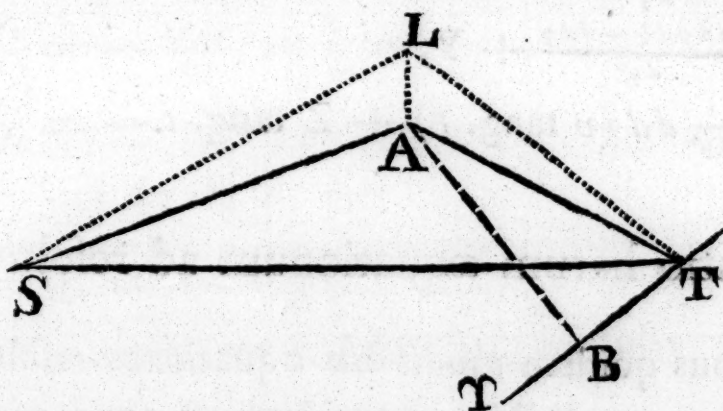
Ubi quidem ob massam terræ respectu Solis fere evanescentem, sine errore statui quoque poterit $\bar{n} = n$, sicque istæ æquationes transformabuntur in has,

$$\text{I. } 0 = \frac{ddv - v d\phi^2}{dq^2} + \frac{g \text{ cof. } l^3}{v^2} + \frac{n^2 v}{u^3} + \frac{n^2}{\pi} \left(\frac{1}{y^2} - \frac{y}{u^3} \right) \text{ cof. } \omega,$$

$$\text{II. } 0 = \frac{2dv d\phi + v dd\phi}{dq^2} - \frac{n^2}{\pi} \left(\frac{1}{y^2} - \frac{y}{u^3} \right) \text{ fin. } \omega,$$

$$\text{III. } 0 = \frac{1}{dq^2} dd(v \text{ tang. } l) + \text{tang. } l \left[\frac{g \text{ cof. } l^3}{v^2} + \frac{n^2 v}{u^3} \right].$$

§ 9. Quoniam in triangulo ATS dantur latera $TA = av$, $ST = \frac{ay}{\pi}$, et angulus interjacens $SAT = \omega$, erit ex trigonometria latus



$$SA = \sqrt{(a^2 v^2 + \frac{a^2 y^2}{\pi^2} - \frac{2a^2 v y}{\pi} \text{ cof. } \omega)};$$

in triangulo autem rectangulo SAL, cognitis SA et LA, erit

$$SL = \sqrt{(SA^2 + a^2 v^2 \text{ tang. } l^2)},$$

hincque SL =

$$\frac{au}{\pi} = \sqrt{\left(\frac{a^2 v^2}{\text{cof. } l^2} + \frac{a^2 y^2}{\pi^2} - \frac{2a^2 v y}{\pi} \text{ cof. } \omega \right)}, \text{ five}$$

$$u = \sqrt{\left(y^2 + \frac{v^2 \pi^2}{\text{cof. } l^2} - 2yv\pi \text{ cof. } \omega \right)}.$$

§ 10. Cum vero in hoc valore ipsius u terminus y^2 præ reliquis admodum magnus sit, dabitur ex theoremate binomiali per approximationem fatis convergentem valor ipsius $\frac{1}{u^3}$, qui proinde evolutus

B

et

et in tribus æquationibus præcedentibus substitutus, rejectis terminis, qui π^2 et altiores hujus potestates involvunt, præbet has

$$\text{I. } 0 = \frac{ddv - v d\phi^2}{dq^2} + \frac{g \cos. l^3}{v^2} - \frac{n^2 v}{2y^3} - \frac{3n^2 v}{2y^3} \cos. 2\omega - \frac{9\pi n^2 v^2}{8y^4} \cos. \omega - \frac{15\pi n^2 v^2}{8y^4} \cos. 3\omega,$$

$$\text{II. } 0 = \frac{2dv d\phi + v dd\phi}{dq^2} + \frac{3n^2 v \sin. 2\omega}{2y^3} + \frac{3\pi n^2 v^2 \sin. \omega}{8y^4} + \frac{15\pi n^2 v^2 \sin. 3\omega}{8y^4},$$

$$\text{III. } 0 = \frac{1}{dq^2} dd(v \tan. l) + \tan. l \left[\frac{g \cos. l^3}{v^2} + \frac{n^2 v}{y^3} + \frac{3\pi n^2 v^2 \cos. \omega}{y^4} \right].$$

§ 11. Ponatur brevitatis causa

$$\frac{g \cos. l^3}{v^2} - \frac{n^2 v}{2y^3} - \frac{3n^2 v \cos. 2\omega}{2y^3} - \frac{9\pi n^2 v^2 \cos. \omega}{8y^4} - \frac{15\pi n^2 v^2 \cos. 3\omega}{8y^4} = X,$$

$$\frac{3n^2 v \sin. 2\omega}{2y^3} + \frac{3\pi n^2 v^2 \sin. \omega}{8y^4} + \frac{15\pi n^2 v^2 \sin. 3\omega}{8y^4} = Y,$$

$$\frac{g \cos. l^3}{v^2} + \frac{n^2 v}{y^3} + \frac{3\pi n^2 v^2 \cos. \omega}{8y^4} = Z,$$

et erunt hæ æquationes resolvendæ,

$$\text{I. } 0 = \frac{ddv - v d\phi^2}{dq^2} + X,$$

$$\text{II. } 0 = \frac{2dv d\phi - dd\phi}{dq^2} + Y,$$

$$\text{III. } 0 = \frac{1}{dq^2} dd(v \tan. l) + Z \tan. l.$$

Præparatio harum æquationum ad resolutionem.

§ 12. Pluribus quidem modis hæ æquationes resolvi possunt, adproximatione tamen adhibita, ut videre est apud eos, qui conatus suos circa hoc idem negotium publicis scriptis exposuerunt; verum nullus horum modorum maximis difficultatibus ac summa molestia non laborat; mihi tentatis pluribus aliis, brevissimus tandem simulac accuratissimus hic visus est, quem hoc loco adumbrabo.

§ 13. Multiplicetur æquatio II. per $v dq$, eaque ita scribatur

$$\frac{2v dv d\phi - v^2 dd\phi}{dq} = -Y v dq,$$

tunc integration instituta habebitur

$$\frac{v^2 d\phi}{dq} = e - \int Y v dq,$$

denotante e quantitatem constantem, cujus valor deinceps definitur.

§ 14. Jam ponatur $v = \frac{1}{x}$, et $\int \frac{Y dq}{x} = P$, ut sit $\frac{d\phi}{e x^2 dq} = 1 - \frac{P}{e}$; et posito denuo $e x^2 dq = dp$, five $dq = \frac{dp}{e x^2}$, erit $\frac{d\phi}{dp} = 1 - \frac{P}{e}$, et II. $dP = \frac{Y dp}{e x^3}$, in qua quidem æquatione post integrationem nulla constans adjici debet, quia ipsa jam in e continetur.

§ 15. Jam cum in æquatione I. dq positum sit constans, ut aliud differentiale pro constanti haberi queat, concipiatur ea scripta hoc modo

$$0 = \frac{1}{dq} d \frac{dv}{dq} - \frac{v d\phi^2}{dq^2} + X;$$

quoniam igitur $v = \frac{1}{x}$, et $dq = \frac{dp}{e x^2}$ (§ 14.) erit

$$dv = -\frac{dx}{x^2}, \text{ et } \frac{dv}{dq} = -\frac{e dx}{dp};$$

ergo, posito nunc dp constanti

$$d \frac{dv}{dq} = -\frac{e ddx}{dp}, \text{ et}$$

$$\frac{1}{dq} d \frac{dv}{dq} = -\frac{e^2 x^2 ddx}{dp^2},$$

porro, ob $v = \frac{1}{x}$ et $dq = \frac{dp}{e x^2}$, erit

$$-\frac{v d\phi^2}{dq^2} = -\frac{e^2 x^3 d\phi^2}{dp^2},$$

qui ergo valores in æquatione I. substituti, præbent

$$-\frac{e^2 x^2 ddx}{dp^2} - \frac{e^2 x^3 d\phi^2}{dp^2} + X = 0,$$

sive restituto pro $\frac{d\phi}{dp}$ ejus valore supra (§ 14.) invento $1 - \frac{P}{e}$ et reductione facta,

$$I. 0 = \frac{ddx}{dp^2} + x - \frac{X}{e^2 x^2} - \frac{2Px}{e} + \frac{P^2 x}{e^2}.$$

§ 16. Æquatio tertia, quæ motum latitudinis Lunæ complectitur, ab aliis hætenus dispesci solita est in duas, è quarum una locum nodi, ex altera vero inclinationem orbitæ versus eclipticam investigare oportuit. Verum cum hoc modo non solum labor præter necessitatem duplicetur, sed et tabulæ inde constructæ ad usum astronomicum minus aptæ sint; quandoquidem hic revera nosse nihil interest,

terest, quo loco nodus hæreat, quantumve Lunæ orbita inclinetur, modo latitudo Lunæ, quacunque demum ratione, innotescat: propterea satius esse duxi ipsam latitudinem, obmissa nodi et inclinationis consideratione, via directâ ex theoria deducere, quod aptissime fieri potest transformando æquationem tertiam in formam commodam hoc modo.

§ 17. Ut in hac æquatione loco dq aliud differentiale poni possit constans, juvat eam representare ita,

$$0 = \frac{1}{e dq} d \left(\frac{\text{tang. } l}{x} \right) + \frac{Z \text{ tang. } l}{e^2},$$

est vero actu differentiando,

$$d \left(\frac{\text{tang. } l}{x} \right) = \frac{1}{x} d \text{ tang. } l - \frac{dx \text{ tang. } l}{x^2},$$

atque adeo ob $e dq = \frac{dp}{x^2}$,

$$\frac{d \left(\frac{\text{tang. } l}{x} \right)}{e dq} = \frac{x d \text{ tang. } l}{dp} - \frac{\text{tang. } l dx}{dp},$$

hinc iterum differentiando posito dp constanti, et delendo terminos se mutuo destruentes,

$$d \left(\frac{d \left(\frac{\text{tang. } l}{x} \right)}{e dq} \right) = \frac{x dd \text{ tang. } l}{dp} - \frac{d dx \text{ tang. } l}{dp},$$

adeoque

$$\frac{1}{e dq} d \left(\frac{d \left(\frac{\text{tang. } l}{x} \right)}{e dq} \right) = \frac{x^3 dd \text{ tang. } l}{dp^2} - \frac{x^2 d dx \text{ tang. } l}{dp^2},$$

hoc valore autem in ipsa tertia æquatione substituto erit

$$\text{III. } 0 = \frac{x dd \text{ tang. } l}{dp^2} - \frac{\text{tang. } l d dx}{dp^2} + \frac{Z \text{ tang. } l}{e^2 x^2}.$$

Multiplicetur jam I. æquatio per $\text{tang. } l$, et productum ad hanc addatur, eritque divisione per x facta

$$\text{III. } \frac{dd \text{ tang. } l}{dp^2} + \text{tang. } l + \frac{(Z - X)}{e^2 x^3} \text{ tang. } l - \frac{2P \text{ tang. } l}{e} + \frac{P^2 \text{ tang. } l}{e^2},$$

quæ æquatio omnino similis est primæ, atque etiam simili modo resolvetur.

§ 18. Jam restituamus valores ipsarum V, X, Y, et Z, supra (§ 14. 11.) definitos, atque habebimus has tres æquationes principales resolvendas,

$$\text{I. } 0 = \frac{ddx}{dp^2} + x - \frac{g \cos. l^3}{e^2} + \frac{n^2}{2e^2 x^3 y^3} + \frac{3n^2 \cos. 2\omega}{2e^2 x^3 y^3} + \frac{9\pi n^2 \cos. \omega}{8e^2 x^4 y^4} + \frac{15\pi n^2 \cos. 3\omega}{8e^2 x^4 y^4} - \frac{2P}{e} + \frac{P^2}{e^2},$$

$$\text{II. } 0 = -\frac{dP}{dp} + \frac{3n^2 \sin. 2\omega}{2e x^4 y^3} + \frac{3\pi n^2 \sin. \omega}{8e x^5 y^4} + \frac{15\pi n^2 \sin. 3\omega}{8e x^5 y^4},$$

$$\text{III. } 0 = \frac{dd \text{ tang. } l}{dp^2} + \text{tang. } l + \text{tang. } l \left[\frac{3n^2}{2e^2 x^4 y^3} + \frac{3n^2 \cos. 2\omega}{2e^2 x^4 y^3} + \frac{33\pi n^2 \cos. \omega}{8e^2 x^5 y^4} + \frac{15\pi n^2 \cos. 3\omega}{8e^2 x^5 y^4} - \frac{2P}{e} + \frac{P^2}{e^2} \right],$$

quibus adjungendæ ex superioribus æquationes secundariae hæ

$$\frac{d\phi}{dp} = 1 - \frac{P}{e},$$

$$\frac{dq}{dp} = \frac{1}{ex^2}.$$

§ 19. Ad has æquationes debito modo resolvendas requiritur ut quantitates variabiles omnes, h. e. ω , x , y , l , et P , per unicam variabilem p saltem proxime expressæ sint, hincque patet adproximatione esse utendum. Ne vero nimis crebro eam repetere cogamur, pro valore ipsius x affumamus hanc seriem indeterminatam,

$$x = 1 - A \cos. \alpha p - B \cos. \beta p - C \cos. \gamma p - \dots$$

pro valore ipsius $\frac{dP}{dp}$ autem hanc

$$\frac{dP}{dp} = a \sin. \alpha p + b \sin. \beta p + c \sin. \gamma p + \dots$$

in quibus seriebus literæ A, B, C, &c. a , b , c , &c. itemque α , β , γ , &c. quantitates denotant constantes ex æquationibus principalibus determinandas. Has autem formulas valoribus quantitatum x et $\frac{dP}{dp}$ convenire cum ex natura problematis, tum inde patet, quod æquationes istæ sufficiant ad determinandas singulas has quantitates arbitrarias, A, B, C, a , b , c , &c. nullusque terminus, qui sit formæ ab hac $A \cos. \alpha p$, five $a \sin. \alpha p$ diversæ, in illis locum habeat, uti resolutione ipsa instituta plenissime cognoscetur.

Reductio et integratio æquationis $\frac{dq}{dp} = \frac{1}{x^2}$.

§ 20. Cum igitur sit ex hypothesi

$$x = 1 - A \cos. \alpha p - B \cos. \beta p - C \cos. \gamma p - " "$$

erit, posito brevitatis causa

$$A \cos. \alpha p + B \cos. \beta p + C \cos. \gamma p + " = Z,$$

$$\frac{1}{x^2} = (1 - Z)^{-2}$$

et quoniam Z respectu unitatis satis exiguum, ut deinceps videbimus, erit juxta theorema binomiale satis accurate

$$\frac{1}{x^2} = 1 + 2Z + 3Z^2 + 4Z^3 + 5Z^4 + " " "$$

unde, restituto valore ipsius Z , et facta reductione debita admittendo ex termino postremo, qui minimus est, ut totus negligi possit, tantum partes $\frac{15}{8}A^4$ et $\frac{5A^4}{8} \cos. 4\alpha p$, quæ reliquarum maximæ sunt, obtinebimus

$$\begin{aligned} \frac{1}{x^2} = & 1 + \frac{3}{2}A^2 + (2A + 3A^3 + 6AB^2 + 6AC^2) \cos. \alpha p + \left(\frac{3}{2}A^2 + \frac{5A^4}{2}\right) \cos. 2\alpha p + 3AB \cos. (\alpha \pm \beta)p, \\ & + \frac{3}{2}B^2 + (2B + 3B^3 + 6BA^2 + 6BC^2) \cos. \beta p + \frac{3B^2}{2} \cos. 2\beta p + 3AC \cos. (\alpha \pm \gamma)p, \\ & + \frac{15}{8}A^4 + (2C + 3C^3 + 6CA^2 + 6CB^2) \cos. \gamma p + \frac{3C^2}{2} \cos. 2\gamma p + 3BC \cos. (\beta \pm \gamma)p, \\ & + A^3 \cos. 3\alpha p + 3A^2B \cos. (2\alpha \pm \beta)p + 6ABC \cos. (\alpha \pm \beta \pm \gamma)p + \frac{5A^4}{8} \cos. 4\alpha p, \\ & + B^3 \cos. 3\beta p + 3A^2C \cos. (2\alpha \pm \gamma)p, \\ & + C^3 \cos. 3\gamma p + 3B^2A \cos. (2\beta \pm \alpha)p, \\ & + 3B^2C \cos. (2\beta \pm \gamma)p, \\ & + 3C^2A \cos. (2\gamma \pm \alpha)p, \\ & + 3C^2B \cos. (2\gamma \pm \beta)p, \end{aligned}$$

in qua forma brevitatis gratia scriptum est $3AB \cos. (\alpha \pm \beta)p$, loco duplicis valoris $3AB \cos. (\alpha + \beta)p$ et $3AB \cos. (\alpha - \beta)p$, ficque etiam in reliquis; terminus autem $6ABC \cos. (\alpha \pm \beta \pm \gamma)p$, quadruplicem valorem designare intelligendus est, hunc in modum $6ABC \cos. (\alpha + \beta + \gamma)p$, $6ABC \cos. (\alpha + \beta - \gamma)p$, $6ABC \cos. (\alpha - \beta + \gamma)p$, $6ABC \cos. (\alpha - \beta - \gamma)p$. Facile autem

autem patet, quam formam reliqui hujus seriei termini induerent, si series pro x adsumpta in plures terminos extenderetur, hoc modo,
 $x = 1 - A \cos. \alpha p - B \cos. \beta p - C \cos. \gamma p - D \cos. \delta p - E \cos. \varepsilon p - \dots$

§ 21. Multiplicatu autem isto valore ipsius $\frac{1}{x^2}$ per $\frac{dp}{e}$, obtinebitur primum, ponendo terminos constantes, qui dp afficiunt, $= 1$, valor ipsius e ex hac æquatione

$$1 = \frac{1 + \frac{3}{2}A^2 + \frac{3}{2}B^2 + \frac{3}{2}C^2 + \frac{15}{8}A^4}{e},$$

adeoque

$$e = 1 + \frac{3}{2}A^2 + \frac{3}{2}B^2 + \frac{3}{2}C^2 + \frac{15}{8}A^4,$$

in qua quidem serie ob parvitatem obmissi sunt termini quarti ordinis, præter hunc $\frac{15}{8}A^4$, qui horum maximus est.

Deinde vero etiam integratione instituta erit

$$\begin{aligned} q = p + & \frac{2A + 3A^3 + 6AB^2 + 6AC^2}{ae} \sin. \alpha p + \frac{3A^2 + 5A^4}{4ae} \sin. 2\alpha p + \frac{3AB}{(a \pm \beta)e} \sin. (\alpha \pm \beta)p, \\ & + \frac{2B + 3B^3 + 6BA^2 + 6BC^2}{\beta e} \sin. \beta p + \frac{3B^2}{4\beta e} \sin. 2\beta p + \frac{3AC}{(a \pm \gamma)e} \sin. (\alpha \pm \gamma)p, \\ & + \frac{2C + 3C^3 + 6CA^2 + 6CB^2}{\gamma e} \sin. \gamma p + \frac{3C^2}{4\gamma e} \sin. 2\gamma p + \frac{3BC}{(\beta \pm \gamma)e} \sin. (\beta \pm \gamma)p, \\ & + \frac{A^3}{3ae} \sin. 3\alpha p + \frac{3A^2B}{(2a \pm \beta)e} \sin. (2\alpha \pm \beta)p + \frac{6ABC}{(a \pm \beta \pm \gamma)e} \sin. (\alpha \pm \beta \pm \gamma)p, \\ & + \frac{B^3}{3\beta e} \sin. 3\beta p + \frac{3A^2C}{(2a \pm \gamma)e} \sin. (2\alpha \pm \gamma)p \left. \begin{aligned} & + \frac{5A^4}{32ae} \sin. 4\alpha p, \\ & + \frac{3B^2C}{(2\beta \pm \gamma)e} \sin. (2\beta \pm \gamma)p, \\ & + \frac{3C^2A}{(2\gamma \pm \alpha)e} \sin. (2\gamma \pm \alpha)p, \\ & + \frac{3C^2B}{(2\gamma \pm \beta)e} \sin. (2\gamma \pm \beta)p. \end{aligned} \right\} \end{aligned}$$

$$\text{Integratio æquationis } \frac{d\phi}{dp} = 1 - \frac{p}{e}.$$

§ 22. Quoniam ex § 19. est

$$\frac{dP}{dp} = a \sin. \alpha p + b \sin. \beta p + c \sin. \gamma p + \dots$$

erit

erit multiplicando per $-\frac{dp}{e}$ et integrando

$$-\frac{p}{e} = \frac{a}{a^2} \cos. \alpha p + \frac{b}{b^2} \cos. \beta p + \frac{c}{c^2} \cos. \gamma p + " "$$

ubi quidem constans nulla adjicienda (§ 14.) quia ipsa jam in e continetur. Hinc igitur est

$$d\varphi = dp + \frac{a}{a^2} \cos. \alpha p dp + \frac{b}{b^2} \cos. \beta p dp + \frac{c}{c^2} \cos. \gamma p dp + " "$$

atque adeo denuo integrando

$$\varphi = p + \frac{a}{a^2} \sin. \alpha p + \frac{b}{b^2} \sin. \beta p + \frac{c}{c^2} \sin. \gamma p + " "$$

Quia ergo per unicam variabilem p dantur valores longitudinis tam veræ φ quam mediæ q , una cum quantitate x , ex qua distantia Lunæ à Terra, five etiam ejusdem parallaxis inveniri poterit; patet totum nunc negotium eo reductum esse, ut valores constantium arbitrariarum $A, B, C, a, b, c, \alpha, \beta, \gamma, \&c.$ convenienti modo determinentur, quod fieri debet ope trium æquationum principalium, quas supra (§ 18.) exhibuimus.

§ 23. Et speciatim quidem secunda harum æquationum inservit ad determinandas quantitates $a, b, c, \&c.$ ex prima autem investigandi sunt valores literarum $A, B, C, \&c.$ verum eadem æquationes conjunctim indicabunt, qui valores coëfficientibus $\alpha, \beta, \gamma, \&c.$ anguli p tribuendi. Tertia æquatio seorsim deinceps tractabitur.

§ 24. In his igitur æquationibus principalibus cum plures variables ab x, y, ω, P, l , pendentes infunt, oportet eas singulas per istam unicam p exprimi; ubi quidem notari convenit, in terminis, qui per quantitatem in se satis parvam n^2 multiplicantur, sufficere, si valores quantitatum ab $x, y, \&c.$ pendentium ad secundam tantum potentiam ipsarum $A, B, a, b, \&c.$ rejectis altioribus, extendantur. Ne tamen ullus scrupulus propter hanc omissionem oriatur, retinebimus ubique maximum eorum terminorum, qui rejiciuntur. Quodsi enim, re peracta, termini, qui hinc in longitudinem Lunæ redundant, minimi inveniantur, tutissime

tutissime inde colligi poterit, neque reliquos, qui ex obmissis oriri potuissent, alicujus momenti esse.

Evolutio terminorum $\frac{1}{x^3}$, $\frac{1}{x^4}$, $\frac{1}{x^5}$.

§ 25. Quæramus primo valores, quantitatum ab x solum pendentium, quem in finem resumamus factam supra (§ 20.) denominationem, ubi posueramus $x = 1 - Z$, eritque ex theoremate binomiali

$$\frac{1}{x^3} = (1 - Z)^{-3} = 1 + 3Z + 6Z^2 + 10Z^3 + \dots$$

$$\frac{1}{x^4} = (1 - Z)^{-4} = 1 + 4Z + 10Z^2 + 20Z^3 + \dots$$

$$\frac{1}{x^5} = (1 - Z)^{-5} = 1 + 5Z + 15Z^2 + \dots$$

reponendo igitur loco Z valorem ejus $A \cos. \alpha p + B \cos. \beta p + C \cos. \gamma p + \dots$ obtinebimus

$$\begin{aligned} \frac{1}{x^3} = e^2 + (3A + \frac{15}{2}A^3) \cos. \alpha p + 3A^2 \cos. 2\alpha p + 6AB \cos. (\alpha \pm \beta)p + \frac{5A^3}{2} \cos. 3\alpha p, \\ + 3B \cos. \beta p + 3B^2 \cos. 2\beta p + 6AC \cos. (\alpha \pm \gamma)p, \\ + 3C \cos. \gamma p + 3C^2 \cos. 2\gamma p + 6BC \cos. (\beta \pm \gamma)p. \end{aligned}$$

$$\begin{aligned} \frac{1}{x^4} = e^{3\frac{1}{2}} + (4A + 15A^3) \cos. \alpha p + 5A^2 \cos. 2\alpha p + 10AB \cos. (\alpha \pm \beta)p + 5A^3 \cos. 3\alpha p, \\ + 4B \cos. \beta p + 5B^2 \cos. 2\beta p + 10AC \cos. (\alpha \pm \gamma)p, \\ + 4C \cos. \gamma p + 5C^2 \cos. 2\gamma p + 10BC \cos. (\beta \pm \gamma)p. \end{aligned}$$

$$\begin{aligned} \frac{1}{x^5} = e^5 + 5A \cos. \alpha p + \frac{15A^2}{2} \cos. 2\alpha p, \\ + 5B \cos. \beta p, \\ + 5C \cos. \gamma p. \end{aligned}$$

Ubi quidem ob $e = 1 + \frac{3}{2}A^2 + \frac{3}{2}B^2 + \dots$ (§ 21.) pro $1 + 3A^2 + 3B^2 + 3C^2 + \dots$ scripsi e^2 , pro $1 + 5A^2 + 5B^2 + 5C^2 + \dots$ autem $e^{3\frac{1}{2}}$, atque e^5 pro $1 + \frac{15A^2}{2} + \frac{15B^2}{2} + \frac{15C^2}{2} + \dots$ quod licet accurate verum non sit, tamen concinnitatis gratia citra erroris metum supponi potest. Valorem ipsius $\frac{1}{x^5}$, qui non nisi terminos per π multiplicatos ingreditur, ad plures terminos extendere inutile est.

Evolutio terminorum $\frac{1}{y^3}$ et $\frac{1}{y^4}$.

§ 26. Posuimus supra (§ 7.) distantiam Solis à Terra mediam $= \frac{\bar{a}}{\pi}$, et distantiam veram (§ 6.) $= \frac{\bar{a}y}{\pi}$, est vero ex theoria Solis, posita anomalia vera Solis $= s$, et eccentricitate orbitæ solaris $= \epsilon$, vera ejus à Terra distantia $= \frac{\bar{a}(1-\epsilon^2)}{\pi(1-\epsilon \cos. s)}$, unde oportet esse $\frac{1}{y} = \frac{1-\epsilon \cos. s}{1-\epsilon^2}$, sive proxime, ob parvitatem ipsius ϵ ,

$$\frac{1}{y} = 1 + \epsilon^2 - \epsilon \cos. s;$$

hinc ergo

$$\frac{1}{y^3} = 1 + \frac{9\epsilon^2}{2} - 3\epsilon \cos. s + \frac{3}{2}\epsilon^2 \cos. 2s,$$

$$\text{et } \frac{1}{y^4} = 1 + 7\epsilon^2 - 4\epsilon \cos. s,$$

qui postremus valor ulteriori continuatione non indiget, quia in terminis per π multiplicatis tantum occurrit.

§ 27. Quoniam igitur ex superioribus (§ 7.) est ratio motus medii Lunæ ad motum medium Solis $1 : \bar{n}$, erit computando locum medium Lunæ q ab apogæo Solis, quod immotum hic statui licet, anomalia media Solis $= \bar{n}q$, ex theoria autem Solis erit proxime

$$s = \bar{n}q - 2\epsilon \sin. \bar{n}q + \frac{5\epsilon^2}{4} \sin. 2\bar{n}q,$$

adeoque

$$\cos. s = \epsilon + \cos. \bar{n}q - \epsilon \cos. 2\bar{n}q + "$$

$$\cos. 2s = \cos. 2\bar{n}q + "$$

qui valores satis accurati in formulis pro $\frac{1}{y^3}$ et $\frac{1}{y^4}$ substituti præbent

$$\frac{1}{y^3} = 1 + \frac{3}{2}\epsilon^2 - 3\epsilon \cos. \bar{n}q + \frac{9\epsilon^2}{2} \cos. 2\bar{n}q,$$

$$\frac{1}{y^4} = 1 + 3\epsilon^2 - 4\epsilon \cos. \bar{n}q.$$

§ 28. Superest ergo, ut in his valoribus etiam $\cos. \bar{n}q$, et $\cos. 2\bar{n}q$ per p exprimantur, quod quidem fiet facile, siquidem ex § 21. valor ipsius q per p datur. Sumamus itaque inde terminos notabiliores,

notabiliores, rejectis iis, quorum nullus hic effectus est, et ponamus

$$\begin{aligned}\bar{n}q &= \bar{n}p + \frac{2A\bar{n}}{a} \sin. \alpha p, \\ &+ \frac{2B\bar{n}}{\beta} \sin. \beta p, \\ &+ \frac{2C\bar{n}}{\gamma} \sin. \gamma p,\end{aligned}$$

critque

$$\begin{aligned}\cos. nq &= \cos. \bar{n}p + \frac{A\bar{n}}{a} \cos. (\alpha + n) p, \\ &- \frac{A\bar{n}}{a} \cos. (\alpha - n) p, \\ &+ \frac{B\bar{n}}{\beta} \cos. (\beta + n) p, \\ &- \frac{B\bar{n}}{\beta} \cos. (\beta - n) p, \\ &+ \frac{C\bar{n}}{\gamma} \cos. (\gamma + n) p, \\ &- \frac{C\bar{n}}{\gamma} \cos. (\gamma - n) p.\end{aligned}$$

Loco $\cos. 2\bar{n}q$ autem sine errore sensibili poni potest simpliciter $\cos. 2\bar{n}p$. Facta igitur horum substitutione obtinebitur

$$\begin{aligned}\frac{1}{y^3} &= 1 + \frac{3}{2}\epsilon^2 - 3\epsilon \cos. \bar{n}p - \frac{3A\bar{n}\epsilon}{a} \cos. (\alpha + n) p + \frac{3\epsilon^2}{2} \cos. 2\bar{n}p, \\ &+ \frac{3A\bar{n}\epsilon}{a} \cos. (\alpha - n) p, \\ &- \frac{3B\bar{n}\epsilon}{\beta} \cos. (\beta + n) p, \\ &+ \frac{3B\bar{n}\epsilon}{\beta} \cos. (\beta - n) p, \\ &- \frac{3C\bar{n}\epsilon}{\gamma} \cos. (\gamma + n) p, \\ &+ \frac{3C\bar{n}\epsilon}{\gamma} \cos. (\gamma - n) p.\end{aligned}$$

$$\frac{1}{y^4} = 1 + 3\epsilon^2 - 4\epsilon \cos. \bar{n}p + " "$$

Evolutio terminorum $\frac{1}{x^3y^3}$, $\frac{1}{x^4y^3}$, $\frac{1}{x^4y^4}$, et $\frac{1}{x^5y^4}$.

§ 29. Facilime jam valores quantitatum $\frac{1}{x^3y^3}$, $\frac{1}{x^4y^3}$, $\frac{1}{x^4y^4}$, et $\frac{1}{x^5y^4}$, inveniri possunt, multiplicando nempe series pro $\frac{1}{x^3}$, $\frac{1}{x^4}$, et $\frac{1}{x^5}$, supra (§ 25.) exhibitas per eas, quas pro $\frac{1}{y^3}$ et $\frac{1}{y^4}$ modo invenimus. Hac ergo

ergo multiplicatione peracta et rejectis terminis et coefficientibus minimis proveniet

$$\begin{aligned} \frac{x}{x^3 y^3} = e^2 (1 + \frac{1}{2} \epsilon^2) + (3A + \frac{1}{2} A^2) (1 + \frac{1}{2} \epsilon^2) \cos. \alpha p + 3A^2 \cos. 2\alpha p + 6AB \cos. (\alpha \pm \beta) p + \frac{15}{2} A^3 \cos. 3\alpha p, \\ + 3B (1 + \frac{3}{2} \epsilon^2) \cos. \beta p + 3B^2 \cos. 2\beta p + 6AC \cos. (\alpha \pm \gamma) p, \\ + 3C (1 + \frac{3}{2} \epsilon^2) \cos. \gamma p + 3C^2 \cos. 2\gamma p + 6BC \cos. (\beta \pm \gamma) p, \\ - 3\epsilon e^2 \cos. \bar{n} p + \frac{9}{2} \epsilon^2 \cos. 2\bar{n} p - (\frac{9}{2} A\epsilon + \frac{3}{2} \frac{A\bar{n}\epsilon}{\alpha}) \cos. (\bar{n} + \alpha) p, \\ - (\frac{9}{2} A\epsilon - \frac{3}{2} \frac{A\bar{n}\epsilon}{\alpha}) \cos. (\bar{n} - \alpha) p, \\ - (\frac{9}{2} B\epsilon + \frac{3}{2} \frac{B\bar{n}\epsilon}{\beta}) \cos. (\bar{n} + \beta) p, \\ - (\frac{9}{2} B\epsilon - \frac{3}{2} \frac{B\bar{n}\epsilon}{\beta}) \cos. (\bar{n} - \beta) p, \\ - (\frac{9}{2} C\epsilon + \frac{3}{2} \frac{C\bar{n}\epsilon}{\gamma}) \cos. (\bar{n} + \gamma) p, \\ - (\frac{9}{2} C\epsilon - \frac{3}{2} \frac{C\bar{n}\epsilon}{\gamma}) \cos. (\bar{n} - \gamma) p. \end{aligned}$$

$$\begin{aligned} \frac{x}{x^4 y^3} = e^{3\frac{1}{2}} (1 + \frac{3}{2} \epsilon^2) + (4A + 15A^2) (1 + \frac{3}{2} \epsilon^2) \cos. \alpha p + 5A^2 \cos. 2\alpha p + 10AB \cos. (\alpha \pm \beta) p + 15A^3 \cos. 3\alpha p, \\ + 4B (1 + \frac{3}{2} \epsilon^2) \cos. \beta p + 5B^2 \cos. 2\beta p + 10AC \cos. (\alpha \pm \gamma) p, \\ + 4C (1 + \frac{3}{2} \epsilon^2) \cos. \gamma p + 5C^2 \cos. 2\gamma p + 10BC \cos. (\beta \pm \gamma) p, \\ - 3\epsilon e^{3\frac{1}{2}} \cos. \bar{n} p + \frac{9}{2} \epsilon^2 \cos. 2\bar{n} p - (6A\epsilon + \frac{3}{2} \frac{A\bar{n}\epsilon}{\alpha}) \cos. (\bar{n} + \alpha) p, \\ - (6A\epsilon - \frac{3}{2} \frac{A\bar{n}\epsilon}{\alpha}) \cos. (\bar{n} - \alpha) p, \\ - (6B\epsilon + \frac{3}{2} \frac{B\bar{n}\epsilon}{\beta}) \cos. (\bar{n} + \beta) p, \\ - (6B\epsilon - \frac{3}{2} \frac{B\bar{n}\epsilon}{\beta}) \cos. (\bar{n} - \beta) p, \\ - (6C\epsilon + \frac{3}{2} \frac{C\bar{n}\epsilon}{\gamma}) \cos. (\bar{n} + \gamma) p, \\ - (6C\epsilon - \frac{3}{2} \frac{C\bar{n}\epsilon}{\gamma}) \cos. (\bar{n} - \gamma) p. \end{aligned}$$

$$\begin{aligned} \frac{x}{x^4 y^4} = e^{3\frac{1}{2}} (1 + 3\epsilon^2) + 4A \cos. \alpha p + 5A^2 \cos. 2\alpha p, \\ + 4B \cos. \beta p, \\ + 4C \cos. \gamma p, \\ - 4\epsilon \cos. \bar{n} p. \end{aligned}$$

$$\begin{aligned} \frac{x}{x^5 y^4} = e^5 (1 + 3\epsilon^2) + 5A \cos. \alpha p + 7\frac{1}{2} A^2 \cos. 2\alpha p, \\ + 5B \cos. \beta p, \\ + 5C \cos. \gamma p, \\ - 4\epsilon \cos. \bar{n} p. \end{aligned}$$

Investigatio

Investigatio valorum $\cos. 2\omega$, $\sin. 2\omega$, $\cos. \omega$, $\sin. \omega$,
 $\cos. 3\omega$, $\sin. 3\omega$.

§ 30. Angulus ω denotat distantiam Lunæ à Sole (§ 6.) inveniturque subtrahendo longitudinem Solis veram à longitudine Lunæ vera. Est autem ex theoria Solis, et § 27. longitudo Solis ab ejus apogæo computata $= \bar{n}q - 2\varepsilon \sin. \bar{n}q + \frac{\varepsilon^2}{4} \sin. 2\bar{n}q$, adeoque si longitudinem veram Lunæ φ indidem computemus

$$\omega = \varphi - \bar{n}q + 2\varepsilon \sin. \bar{n}q - \frac{\varepsilon^2}{4} \sin. 2\bar{n}q,$$

quia vero ex § 28. $\bar{n}q = \bar{n}p + \frac{2A\bar{n}}{a} \sin. \alpha p$, proxime

$$+ \frac{2B\bar{n}}{\beta} \sin. \beta p,$$

$$+ \frac{2C\bar{n}}{\gamma} \sin. \gamma p,$$

$$\text{erit } \sin. \bar{n}q = \sin. \bar{n}p + \frac{A\bar{n}}{a} \sin. (n + \alpha) p,$$

$$- \frac{A\bar{n}}{a} \sin. (n - \alpha) p,$$

$$+ \frac{B\bar{n}}{\beta} \sin. (n + \beta) p,$$

$$- \frac{B\bar{n}}{\beta} \sin. (n - \beta) p,$$

$$+ \frac{C\bar{n}}{\gamma} \sin. (n + \gamma) p,$$

$$- \frac{C\bar{n}}{\gamma} \sin. (n - \gamma) p,$$

et $\sin. 2\bar{n}q = \sin. 2\bar{n}p$, hincque

$$\omega = \varphi - \bar{n}q + 2\varepsilon \sin. \bar{n}p + \frac{2A\bar{n}\varepsilon}{a} \sin. (n + \alpha) p - \frac{\varepsilon^2}{4} \sin. 2\bar{n}p,$$

$$- \frac{2A\bar{n}\varepsilon}{a} \sin. (n - \alpha) p,$$

$$+ \frac{2B\bar{n}\varepsilon}{\beta} \sin. (n + \beta) p,$$

$$- \frac{2B\bar{n}\varepsilon}{\beta} \sin. (n - \beta) p,$$

$$+ \frac{2C\bar{n}\varepsilon}{\gamma} \sin. (n + \gamma) p,$$

$$- \frac{2C\bar{n}\varepsilon}{\gamma} \sin. (n - \gamma) p.$$

§ 31. Substituantur loco ϕ et q eorum valores ex § 21 et 22. et ponatur brevitatis causa

$$1 - \bar{n} = r,$$

$$\frac{a}{a^2 e} - \frac{2 A \bar{n}}{a e} - \frac{3 A^3 \bar{n}}{a e} = \dot{a},$$

$$\frac{b}{\beta^2 e} - \frac{2 B \bar{n}}{\beta e} = \dot{b},$$

$$\frac{c}{\gamma^2 e} - \frac{2 C \bar{n}}{\gamma e} = \dot{c},$$

eritque obmissis terminis minimis

$$\begin{aligned} \omega = r p + \dot{a} \sin. \alpha p - \frac{3 A^2 \bar{n}}{4 a} \sin. 2 \alpha p - \frac{3 A B \bar{n}}{a \pm \beta} \sin. (\alpha \pm \beta) p - \frac{A^3 \bar{n}}{3 a} \sin. 3 \alpha p, \\ + \dot{b} \sin. \beta p - \frac{3 B^2 \bar{n}}{4 \beta} \sin. 2 \beta p - \frac{3 A C \bar{n}}{a \pm \gamma} \sin. (\alpha \pm \gamma) p, \\ + \dot{c} \sin. \gamma p - \frac{3 C^2 \bar{n}}{4 \gamma} \sin. 2 \gamma p - \frac{3 B C \bar{n}}{\beta \pm \gamma} \sin. (\beta \pm \gamma) p, \\ + 2 \dot{e} \sin. \bar{n} p - \frac{5 \dot{e}^2}{4} \sin. 2 \bar{n} p + \frac{2 A \bar{n} \dot{e}}{a} \sin. (\bar{n} + \alpha) p, \\ - \frac{2 A \bar{n} \dot{e}}{a} \sin. (\bar{n} - \alpha) p, \\ + \frac{2 B \bar{n} \dot{e}}{\beta} \sin. (\bar{n} + \beta) p, \\ - \frac{2 B \bar{n} \dot{e}}{\beta} \sin. (\bar{n} - \beta) p, \\ + \frac{2 C \bar{n} \dot{e}}{\gamma} \sin. (\bar{n} + \gamma) p, \\ - \frac{2 C \bar{n} \dot{e}}{\gamma} \sin. (\bar{n} - \gamma) p. \end{aligned}$$

§ 32. Facile hinc invenitur fore

$$\begin{aligned} \text{Cof. } 2 \omega = (1 - 4 \dot{e}^2) \text{cof. } 2 r p + 2 \dot{e} \text{cof. } (2 r + \bar{n}) p + \frac{5}{2} \dot{e}^2 \text{cof. } (2 r + 2 \bar{n}) p + \left[\frac{2 A \bar{n} \dot{e}}{a} + 2 \dot{a} \dot{e} \right] \text{cof. } (2 r + \bar{n} + \alpha) p, \\ - 2 \dot{e} \text{cof. } (2 r - \bar{n}) p + \frac{5}{2} \dot{e}^2 \text{cof. } (2 r - 2 \bar{n}) p - \left[\frac{2 A \bar{n} \dot{e}}{a} + 2 \dot{a} \dot{e} \right] \text{cof. } (2 r + \bar{n} - \alpha) p, \\ + \dot{a} \text{cof. } (2 r + \alpha) p + \left[\frac{1}{2} \dot{a}^2 - \frac{3 A^2 \bar{n}}{4 a} \right] \text{cof. } (2 r + 2 \alpha) p + \left[\frac{2 A \bar{n} \dot{e}}{a} - 2 \dot{a} \dot{e} \right] \text{cof. } (2 r - \bar{n} + \alpha) p, \\ - \dot{a} \text{cof. } (2 r - \alpha) p + \left[\frac{1}{2} \dot{a}^2 + \frac{3 A^2 \bar{n}}{4 a} \right] \text{cof. } (2 r - 2 \alpha) p - \left[\frac{2 A \bar{n} \dot{e}}{a} - 2 \dot{a} \dot{e} \right] \text{cof. } (2 r - \bar{n} - \alpha) p, \\ + \dot{b} \text{cof. } (2 r + \beta) p + \left[\frac{1}{2} \dot{b}^2 - \frac{3 B^2 \bar{n}}{4 \beta} \right] \text{cof. } (2 r + 2 \beta) p, \quad \&c. \quad \&c. \\ - \dot{b} \text{cof. } (2 r - \beta) p + \left[\frac{1}{2} \dot{b}^2 + \frac{3 B^2 \bar{n}}{4 \beta} \right] \text{cof. } (2 r - 2 \beta) p - \left[\frac{3 A B \bar{n}}{a + \beta} - \dot{a} \dot{b} \right] \text{cof. } (2 r + \alpha + \beta) p, \\ \&c. \quad \&c. \quad \&c. \quad \&c. \quad - \left[\frac{3 A B \bar{n}}{a - \beta} - \dot{a} \dot{b} \right] \text{cof. } (2 r + \alpha - \beta) p, \\ + \left[\frac{3 A B \bar{n}}{a - \beta} - \dot{a} \dot{b} \right] \text{cof. } (2 r - \alpha + \beta) p, \\ + \left[\frac{3 A B \bar{n}}{a + \beta} + \dot{a} \dot{b} \right] \text{cof. } (2 r - \alpha - \beta) p, \\ \&c. \quad \&c. \\ + \frac{A^3 \bar{n}}{3 a} \text{cof. } (2 r + 3 \alpha) p, \\ + \frac{A^3 \bar{n}}{3 a} \text{cof. } (2 r - 3 \alpha) p. \end{aligned}$$

Cui

Cui plane similis est valor ipsius $\sin. 2\omega$; scilicet,

$$\begin{aligned} \sin. 2\omega = & t(1-4\epsilon^2) \sin. 2rp + 2\epsilon \sin. (2r+n)p + \frac{2}{3}\epsilon^2 \sin. (2r+2n)p + \left[\frac{2Ane}{a} + 2\dot{a}\epsilon\right] \sin. (2r+n+a)p, \\ & - 2\epsilon \sin. (2r-n)p + \frac{2}{3}\epsilon^2 \sin. (2r-2n)p - \left[\frac{2Ane}{a} + 2\dot{a}\epsilon\right] \sin. (2r+n-a)p, \\ & + \dot{a} \sin. (2r+a)p + \left[\frac{1}{2}\dot{a}^2 - \frac{3A^2n}{4a}\right] \sin. (2r+2a)p + \left[\frac{2Ane}{a} - 2\dot{a}\epsilon\right] \sin. (2r-n+a)p, \\ & - \dot{a} \sin. (2r-a)p + \left[\frac{1}{2}\dot{a}^2 + \frac{3A^2n}{4a}\right] \sin. (2r-2a)p - \left[\frac{2Ane}{a} - 2\dot{a}\epsilon\right] \sin. (2r-n-a)p, \\ & + \dot{b} \sin. (2r+\beta)p + \left[\frac{1}{2}\dot{b}^2 - \frac{3B^2n}{4\beta}\right] \sin. (2r+2\beta)p, \quad \&c. \quad \&c. \\ & - \dot{b} \sin. (2r-\beta)p + \left[\frac{1}{2}\dot{b}^2 + \frac{3B^2n}{4\beta}\right] \sin. (2r-2\beta)p - \left[\frac{3ABn}{a+\beta} - \dot{a}\dot{b}\right] \sin. (2r+\alpha+\beta)p, \\ & \&c. \quad \&c. \quad \&c. \quad \&c. \quad - \left[\frac{3ABn}{a-\beta} + \dot{a}\dot{b}\right] \sin. (2r+\alpha-\beta)p, \\ & + \left[\frac{3ABn}{a-\beta} - \dot{a}\dot{b}\right] \sin. (2r-\alpha+\beta)p, \\ & + \left[\frac{3ABn}{a+\beta} + \dot{a}\dot{b}\right] \sin. (2r-\alpha-\beta)p, \\ & \&c. \quad \&c. \\ & - \frac{A^3n}{3a} \sin. (2r+3a)p, \\ & + \frac{A^3n}{3a} \sin. (2r-3a)p. \end{aligned}$$

Et si vero in his formulis non omnes termini, quos combinatio angularum $2rp$, αp , βp , γp , np , præbet, expressi sint, facile tamen ex his, quos adposui, eorum forma colligi potest; brevitatis autem causa literam t adhibui ad designandam hanc seriem

$$1 - \dot{a}^2 - \dot{b}^2 - \dot{c}^2 - \dots$$

§ 33. Porro autem invenietur satis accurate

$$\begin{aligned} \cos. \omega = & \cos. rp + \epsilon \cos. (r+n)p - \frac{3A^2n}{8a} \cos. (r+2a)p, \\ & - \epsilon \cos. (r-n)p \left\{ + \frac{3A^2n}{8a} \cos. (r-2a)p, \right. \\ & + \frac{1}{2}\dot{a} \cos. (r+\alpha)p \left\{ \right. \\ & - \frac{1}{2}\dot{a} \cos. (r-\alpha)p, \\ & + \frac{1}{2}\dot{b} \cos. (r+\beta)p, \\ & - \frac{1}{2}\dot{b} \cos. (r-\beta)p, \\ & \&c. \quad \&c. \end{aligned}$$

Sin.

$$\begin{aligned}
\text{Sin. } \omega = & \text{fin. } rp + \varepsilon \text{ fin. } (r + n) p - \frac{3A^2n}{8a} \text{ fin. } (r + 2\alpha) p, \\
& - \varepsilon \text{ fin. } (r - n) p \left\{ + \frac{3A^2n}{8a} \text{ fin. } (r - 2\alpha) p, \right. \\
& + \frac{3}{2}a \text{ fin. } (r + \alpha) p \left. \right\} \\
& - \frac{3}{2}a \text{ fin. } (r - \alpha) p, \\
& + \frac{3}{2}b \text{ fin. } (r + \beta) p, \\
& - \frac{3}{2}b \text{ fin. } (r - \beta) p, \\
& \&c. \quad \&c.
\end{aligned}$$

$$\begin{aligned}
\text{Cof. } 3\omega = & \text{cof. } 3rp + 3\varepsilon \text{ cof. } (3r + n) p - \frac{9A^2n}{8a} \text{ cof. } (3r + 2\alpha) p, \\
& - 3\varepsilon \text{ cof. } (3r - n) p \left\{ + \frac{9A^2n}{8a} \text{ cof. } (3r - 2\alpha) p, \right. \\
& + \frac{3}{2}a \text{ cof. } (3r + \alpha) p \left. \right\} \\
& - \frac{3}{2}a \text{ cof. } (3r - \alpha) p, \\
& + \frac{3}{2}b \text{ cof. } (3r + \beta) p, \\
& - \frac{3}{2}b \text{ cof. } (3r - \beta) p, \\
& \&c. \quad \&c.
\end{aligned}$$

$$\begin{aligned}
\text{Sin. } 3\omega = & \text{fin. } 3rp + 3\varepsilon \text{ fin. } (3r + n) p - \frac{9A^2n}{8a} \text{ fin. } (3r + 2\alpha) p, \\
& - 3\varepsilon \text{ fin. } (3r - n) p \left\{ + \frac{9A^2n}{8a} \text{ fin. } (3r - 2\alpha) p, \right. \\
& + \frac{3}{2}a \text{ fin. } (3r + \alpha) p \left. \right\} \\
& - \frac{3}{2}a \text{ fin. } (3r - \alpha) p, \\
& + \frac{3}{2}b \text{ fin. } (3r + \beta) p, \\
& - \frac{3}{2}b \text{ fin. } (3r - \beta) p, \\
& \&c. \quad \&c.
\end{aligned}$$

Neque in his necessum esse existimavi, ut terminos eos, quorum formam ex adscriptis quivis facile adgnosceret, adponerem. Ceterum termini in his seriebus postremum locum occupantes; scilicet, qui in valore cof. 2ω et fin. 2ω coefficientem habent A^3 ; in valore cof. ω , fin. ω , cof. 3ω , et fin. 3ω autem coefficientem A^2 , juxta ea, quæ supra (§ 24.) innui, ideo retinui, quia obmissorum maximi sunt; infra enim videbimus, A esse, saltem posse esse, excentricitatem or-

bitæ lunaris, et maximam omnium quantitatum sub literis A, B, C, a, b, c, et e, intelligendarum.

Evolutio terminorum $-\frac{2Px}{e} + \frac{P^2x}{e^2}$.

§ 34. Supra jam (§ 22.) invenimus esse

$$-\frac{P}{e} = \frac{a}{ae} \cos. \alpha p + \frac{b}{\beta e} \cos. \beta p + \frac{c}{\gamma e} \cos. \gamma p + "$$

et cum sit $x = 1 - A \cos. \alpha p - B \cos. \beta p - C \cos. \gamma p - "$

facile hinc, multiplicatione rite instituta, colligitur fore

$$\begin{aligned} -\frac{2Px}{e} + \frac{P^2x}{e^2} = & -\frac{Aa}{ae} - \frac{Bb}{\beta e} - \frac{Cc}{\gamma e} + \frac{a^2}{2a^2e^2} + \frac{b^2}{2\beta^2e^2} + \frac{c^2}{2\gamma^2e^2}, \\ & \left[+\frac{2a}{ae} - A \left(\frac{3a^2}{4a^2e^2} + \frac{b^2}{2\beta^2e^2} + \frac{c^2}{2\gamma^2e^2} \right) - \frac{Bab}{a\beta e^2} - \frac{Cac}{a\gamma e^2} \right] \cos. \alpha p, \\ & \left[+\frac{2b}{\beta e} - B \left(\frac{3b^2}{4\beta^2e^2} + \frac{a^2}{2a^2e^2} + \frac{c^2}{2\gamma^2e^2} \right) - \frac{Aab}{a\beta e^2} - \frac{Cbc}{\beta\gamma e^2} \right] \cos. \beta p, \\ & \left[+\frac{2c}{\gamma e} - C \left(\frac{3c^2}{4\gamma^2e^2} + \frac{a^2}{2a^2e^2} + \frac{b^2}{2\beta^2e^2} \right) - \frac{Bbc}{\beta\gamma e^2} - \frac{Aac}{a\gamma e^2} \right] \cos. \gamma p, \\ & + \left[\frac{a^2}{2a^2e^2} - \frac{Aa}{ae} \right] \cos. 2\alpha p + \left[\frac{ab}{a\beta e^2} - \frac{Ab}{\beta e} - \frac{Ba}{ae} \right] \cos. (\alpha \pm \beta)p - \frac{Aa^2}{4a^2e^2} \cos. 3\alpha p, \\ & + \left[\frac{b^2}{2\beta^2e^2} - \frac{Bb}{\beta e} \right] \cos. 2\beta p + \left[\frac{ac}{a\gamma e^2} - \frac{Ac}{\gamma e} - \frac{Ca}{ae} \right] \cos. (\alpha \pm \gamma)p - \frac{Bb^2}{4\beta^2e^2} \cos. 3\beta p, \\ & + \left[\frac{c^2}{2\gamma^2e^2} - \frac{Cc}{\gamma e} \right] \cos. 2\gamma p + \left[\frac{bc}{\beta\gamma e^2} - \frac{Bc}{\gamma e} - \frac{Cb}{\beta e} \right] \cos. (\beta \pm \gamma)p - \frac{Cc^2}{4\gamma^2e^2} \cos. 3\gamma p, \\ & - \left[\frac{Aab}{2a\beta e^2} + \frac{Ba^2}{2a^2e^2} \right] \cos. (2\alpha \pm \beta)p + \left[\frac{Abc}{2\beta\gamma e^2} + \frac{Bac}{2a\gamma e^2} + \frac{Cab}{2a\beta e^2} \right] \cos. (\alpha \pm \beta \pm \gamma)p, \\ & - \left[\frac{Aac}{2a\gamma e^2} + \frac{Ca^2}{2a^2e^2} \right] \cos. (2\alpha \pm \gamma)p, \\ & - \left[\frac{Rba}{2\beta a e^2} + \frac{Ab^2}{2\beta^2e^2} \right] \cos. (2\beta \pm \alpha)p, \\ & - \left[\frac{Bbc}{2\beta\gamma e^2} + \frac{Cb^2}{2\beta^2e^2} \right] \cos. (2\beta \pm \gamma)p, \\ & - \left[\frac{Cca}{2\gamma a e^2} + \frac{Ac^2}{2\gamma^2e^2} \right] \cos. (2\gamma \pm \alpha)p, \\ & - \left[\frac{Ccb}{2\gamma\beta e^2} + \frac{Bc^2}{2\gamma^2e^2} \right] \cos. (2\gamma \pm \beta)p, \end{aligned}$$

de signis duplicibus $\pm$ eadem tenenda, quæ supra ad finem § 20. monita sunt.

Valor termini $-\frac{g \cos. l^3}{e^2}$.

§ 35. Valor hujus termini $\frac{g \cos. l^3}{e^2}$ in æquatione principali I substituentus supponit cognitum valorem tang l , atque adeo resolutam æquationem principalem tertiam; neque tamen eam accuratissime resolutam requirit, cum valor cos. l satis prope cognosci queat, si tang. l non sit nimis magnus, quod commodum in Luna locum habet, atque primarii tantum ejus termini undecunque cogniti habeantur. Ne igitur resolutio æquationum nostrarum alias satis molesta ac difficilis, magis adhuc implicetur, ex calculis meis prioribus supponam hunc valorem pro $-\frac{g \cos. l^3}{e^2}$,

$$\begin{aligned}
 -\frac{g \cos. l^3}{e^2} = & -0,9939656 \frac{g}{e^2} - 60130^{\text{vii}} \frac{g}{e^2} \cos. 2ip & + 3600 \cos. (2r - 2i)p, \\
 & + 3516 \cos. 2rp & - 167 \cos. (2r + 2i)p, \\
 & + 6 \cos. np & + 150 \cos. (2i + n)p, \\
 & - 86 \cos. \alpha p & - 159 \cos. (2i - n)p, \\
 & - 72 \cos. (2r + n)p & - 47 \cos. (2i + \alpha)p, \\
 & + 186 \cos. (2r - n)p & + 141 \cos. (2i - \alpha)p, \\
 & + 36 \cos. (2r + \alpha)p & + 83 \cos. (4r - 4i)p, \\
 & - 126 \cos. (2r - \alpha)p & - 50 \cos. (4r - 2i)p, \\
 & - 186 \cos. 2\alpha p & + 80 \cos. (2r - 2i + n)p, \\
 & + 43 \cos. (2r - 2\alpha)p & - 204 \cos. (2r - 2i - n)p, \\
 & - 11 \cos. 4rp & - 39 \cos. (2r - 2i + \alpha)p, \\
 & - 21 \cos. (4r - 2\alpha)p & + 220 \cos. (2r - 2i - \alpha)p, \\
 & - 155 \cos. 4ip & \left. \begin{array}{l} \\ \\ \end{array} \right\} - 274 \cos. (2r - 2\alpha + 2i)p, \\
 & - 3 \cos. rp & \\
 & + 2 \cos. (\alpha + n)p & + 237 \cos. (2r - 2\alpha - 2i)p, \\
 & + 6 \cos. (\alpha - n)p & + 200 \cos. (2\alpha - 2i)p,
 \end{aligned}$$

est autem in hac forma $i:1$ ratio motus Lunæ à nodo ad motum longitudinis; et numeri sunt partes decimales septimæ, hoc est tales, qualium

qualium unitas continet 10000000. $\alpha : 1$ vero est ratio motus anomalie ad motum longitudinis.

Transformatio æquationum I et II.

§ 36. Quodsi jam valores à § 29. hactenus inventi substituantur in æquationibus nostris principalibus I et II. (§ 18.) hæc earum erit forma:

I. Æquatio $0 =$

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ter-
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rum.

1	$+A[\alpha^2 - 1 + \frac{3n^2}{2e^2}(1 + \frac{3}{2}\epsilon^2)(1 + \frac{5}{2}A^2) - \frac{3a^2}{4a^2e^2} - \frac{b^2}{2\beta^2e^2} - \frac{c^2}{2\gamma^2e^2}] + \frac{2a}{ae} - \frac{Bab}{a\beta e^2} - \frac{Cac}{a\gamma e^2} - 86 \text{ cof. } \alpha p$
2	$+B[\beta^2 - 1 + \frac{3n^2}{2e^2}(1 + \frac{3}{2}\epsilon^2) - \frac{3b^2}{4\beta^2e^2} - \frac{a^2}{2a^2e^2} - \frac{c^2}{2\gamma^2e^2}] + \frac{2b}{\beta e} - \frac{Aab}{a\beta e^2} - \frac{Cbc}{\beta\gamma e^2} \text{ cof. } \beta p$
3	$+C[\gamma^2 - 1 + \frac{3n^2}{2e^2}(1 + \frac{3}{2}\epsilon^2) - \frac{3c^2}{4\gamma^2e^2} - \frac{a^2}{2a^2e^2} - \frac{b^2}{2\beta^2e^2}] + \frac{2c}{\gamma e} - \frac{Bbc}{\beta\gamma e^2} - \frac{Aac}{a\gamma e^2} \text{ cof. } \gamma p$
4	$+t(1 - 2\frac{1}{2}\epsilon^2)\frac{3}{2}n^2 - 3516 \dots \dots \dots \text{ cof. } 2rp$
5	$\frac{3}{2}n^2[+\dot{a}(1 + \frac{3}{2}\epsilon^2) + (\frac{3A}{2} + \frac{15A^3}{4})(1 - 2\frac{1}{2}\epsilon^2)\frac{t}{e^2} + \frac{3A\dot{a}^2}{4} - \frac{9A^3n}{8a} - \frac{3A^2\dot{a}}{2}] + 36 \text{ cof. } 2(r + \alpha)p$
6	$\frac{3}{2}n^2[-\dot{a}(1 + \frac{3}{2}\epsilon^2) + (\frac{3A}{2} + \frac{15A^3}{4})(1 - 2\frac{1}{2}\epsilon^2)\frac{t}{e^2} + \frac{3A\dot{a}^2}{4} + \frac{9A^3n}{8a} + \frac{3A^2\dot{a}}{2}] - 126 \text{ cof. } (2r - \alpha)p$
7	$\frac{3}{2}n^2[+\dot{b}(1 + \frac{3}{2}\epsilon^2) + \frac{3}{2}B(1 - 2\frac{1}{2}\epsilon^2)\frac{t}{e^2}] \dots \dots \dots \text{ cof. } (2r + \beta)p$
8	$\frac{3}{2}n^2[-\dot{b}(1 + \frac{3}{2}\epsilon^2) + \frac{3}{2}B(1 - 2\frac{1}{2}\epsilon^2)\frac{t}{e^2}] \dots \dots \dots \text{ cof. } (2r - \beta)p$
9	$\frac{3}{2}n^2[+\dot{c}(1 + \frac{3}{2}\epsilon^2) + \frac{3}{2}C(1 - 2\frac{1}{2}\epsilon^2)\frac{t}{e^2}] \dots \dots \dots \text{ cof. } (2r + \gamma)p$
10	$\frac{3}{2}n^2[-\dot{c}(1 + \frac{3}{2}\epsilon^2) + \frac{3}{2}C(1 - 2\frac{1}{2}\epsilon^2)\frac{t}{e^2}] \dots \dots \dots \text{ cof. } (2r - \gamma)p$
11	$\frac{3}{2}n^2[+2\epsilon(1 + \frac{3}{2}\epsilon^2) - \frac{3}{2}\epsilon(1 - 4\epsilon^2)\frac{t}{e^2}] - 72 \dots \dots \dots \text{ cof. } (2r + n)p$
12	$\frac{3}{2}n^2[-2\epsilon(1 + \frac{3}{2}\epsilon^2) - \frac{3}{2}\epsilon(1 - 4\epsilon^2)\frac{t}{e^2}] + 186 \dots \dots \dots \text{ cof. } (2r - n)p$
13	$\frac{3}{2}n^2[+\frac{3}{2}A^2 + \frac{3}{2}A\dot{a} + \frac{1}{2}\dot{a}^2 - \frac{3A^2n}{4a}] \dots \dots \dots \text{ cof. } (2r + 2\alpha)p$
14	$\frac{3}{2}n^2[+\frac{3}{2}A^2 - \frac{3}{2}A\dot{a} + \frac{1}{2}\dot{a}^2 + \frac{3A^2n}{4a}] + 43 \dots \dots \dots \text{ cof. } (2r - 2\alpha)p$
15	$\frac{3}{2}n^2[+\frac{3}{2}B^2 + \frac{3}{2}B\dot{b} + \frac{1}{2}\dot{b}^2 - \frac{3B^2n}{4\beta}] \dots \dots \dots \text{ cof. } (2r + 2\beta)p$
16	$\frac{3}{2}n^2[+\frac{3}{2}B^2 - \frac{3}{2}B\dot{b} + \frac{1}{2}\dot{b}^2 + \frac{3B^2n}{4\beta}] \dots \dots \dots \text{ cof. } (2r - 2\beta)p$

I. Aequatio. $\odot =$ Ordo
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41	$\frac{3}{2}n^2 \left[+ 3BC + \frac{3B\dot{c}}{2} + b\dot{c} + \frac{3C\dot{b}}{2} - \frac{3BCn}{\beta + \gamma} \right]$	$\text{cof. } (2r + \beta + \gamma)p$
42	$\frac{3}{2}n^2 \left[+ 3BC - \frac{3B\dot{c}}{2} - b\dot{c} + \frac{3C\dot{b}}{2} - \frac{3BCn}{\beta - \gamma} \right]$	$\text{cof. } (2r + \beta - \gamma)p$
43	$\frac{3}{2}n^2 \left[+ 3BC + \frac{3B\dot{c}}{2} - b\dot{c} - \frac{3C\dot{b}}{2} + \frac{3BCn}{\beta - \gamma} \right]$	$\text{cof. } (2r - \beta + \gamma)p$
44	$\frac{3}{2}n^2 \left[+ 3BC - \frac{3B\dot{c}}{2} + b\dot{c} - \frac{3C\dot{b}}{2} + \frac{3BCn}{\beta + \gamma} \right]$	$\text{cof. } (2r - \beta - \gamma)p$
45	$\frac{3}{2}n^2 \left[+ \frac{5A^3}{4} - \frac{35A^3n}{24a} + \frac{3A^2\dot{a}}{2} + \frac{3A\dot{a}^2}{4} \right]$	$\text{cof. } (2r + 3\alpha)p$
46	$\frac{3}{2}n^2 \left[+ \frac{5A^3}{4} + \frac{35A^3n}{24a} - \frac{3A^2\dot{a}}{2} + \frac{3A\dot{a}^2}{4} \right]$	$\text{cof. } (2r - 3\alpha)p$
47	$+ \frac{9\pi n^2}{8} [1 + 3\varepsilon^2] e^{i\frac{1}{2}} - 3 \dots \dots \dots$	$\text{cof. } rp$
48	$\frac{9\pi n^2}{8} [+ 2A + \frac{1}{2}\dot{a}] \dots \dots \dots$	$\text{cof. } (r + \alpha)p$
49	$\frac{9\pi n^2}{8} [+ 2A - \frac{1}{2}\dot{a}] \dots \dots \dots$	$\text{cof. } (r - \alpha)p$
50	$\frac{9\pi n^2}{8} [+ 2B + \frac{1}{2}\dot{b}] \dots \dots \dots$	$\text{cof. } (r + \beta)p$
51	$\frac{9\pi n^2}{8} [+ 2B - \frac{1}{2}\dot{b}] \dots \dots \dots$	$\text{cof. } (r - \beta)p$
52	$\frac{9\pi n^2}{8} [+ 2C + \frac{1}{2}\dot{c}] \dots \dots \dots$	$\text{cof. } (r + \gamma)p$
53	$\frac{9\pi n^2}{8} [+ 2C - \frac{1}{2}\dot{c}] \dots \dots \dots$	$\text{cof. } (r - \gamma)p$
54	$\frac{9\pi n^2}{8} [-\varepsilon] \dots \dots \dots$	$\text{cof. } (r + n)p$
55	$\frac{9\pi n^2}{8} [-3\varepsilon] \dots \dots \dots$	$\text{cof. } (r - n)p$
56	$\frac{9\pi n^2}{8} [+ \frac{5A^2}{2} - \frac{3A^2n}{8} + A\dot{a}] \dots \dots \dots$	$\text{cof. } (r + 2\alpha)p$
57	$\frac{9\pi n^2}{8} [+ \frac{5A^2}{2} + \frac{3A^2n}{8} - A\dot{a}] \dots \dots \dots$	$\text{cof. } (r - 2\alpha)p$
58	$+ \frac{15\pi n^2}{8} [1 + 3\varepsilon^2] e^{i\frac{1}{2}} \dots \dots \dots$	$\text{cof. } 3rp$
59	$\frac{15\pi n^2}{8} [+ 2A + \frac{3}{2}\dot{a}] \dots \dots \dots$	$\text{cof. } (3r + \alpha)p$
60	$\frac{15\pi n^2}{8} [+ 2A - \frac{3}{2}\dot{a}] \dots \dots \dots$	$\text{cof. } (3r - \alpha)p$
61	$\frac{15\pi n^2}{8} [+ 2B + \frac{3}{2}\dot{b}] \dots \dots \dots$	$\text{cof. } (3r + \beta)p$
62	$\frac{15\pi n^2}{8} [+ 2B - \frac{3}{2}\dot{b}] \dots \dots \dots$	$\text{cof. } (3r - \beta)p$
63	$\frac{15\pi n^2}{8} [+ 2C + \frac{3}{2}\dot{c}] \dots \dots \dots$	$\text{cof. } (3r + \gamma)p$
64	$\frac{15\pi n^2}{8} [+ 2C - \frac{3}{2}\dot{c}] \dots \dots \dots$	$\text{cof. } (3r - \gamma)p$

I. Æquatio. 0 =

Ordo
terminorum.

65	$\frac{15\pi n^2}{8} [+ \varepsilon]$	cos. $(3r+n)p$
66	$\frac{15\pi n^2}{8} [- 5\varepsilon]$	cos. $(3r-n)p$
67	$\frac{15\pi n^2}{8} [+ \frac{5A^2}{2} - \frac{9A^2n}{8} + 3A\dot{a}]$	cos. $(3r+2\alpha)p$
68	$\frac{15\pi n^2}{8} [+ \frac{5A^2}{2} + \frac{9A^2n}{8} - 3A\dot{a}]$	cos. $(3r-2\alpha)p$
69	$-0,9939656 \frac{r}{e^2} + \frac{n^2}{2} (1 + \frac{3}{2}\varepsilon^2) - \frac{Aa}{ae} - \frac{Bb}{\beta e} - \frac{Cc}{\gamma e} + \frac{a^2}{2a^2e^2} + \frac{b^2}{2\beta^2e^2} + \frac{c^2}{2\gamma^2e^2}$	constantes
70	$+ \frac{3}{2}n^2 \cdot 2AB + \frac{ab}{a\beta e^2} - \frac{Ab}{\beta e} - \frac{Ba}{ae}$	cos. $(\alpha + \beta)p$
71	$+ \frac{3}{2}n^2 \cdot 2AB + \frac{ab}{a\beta e^2} - \frac{Ab}{\beta e} - \frac{Ba}{ae}$	cos. $(\alpha - \beta)p$
72	$+ \frac{3}{2}n^2 \cdot 2AC + \frac{ac}{a\gamma e^2} - \frac{Ac}{\gamma e} - \frac{Ca}{ae}$	cos. $(\alpha + \gamma)p$
73	$+ \frac{3}{2}n^2 \cdot 2AC + \frac{ac}{a\gamma e^2} - \frac{Ac}{\gamma e} - \frac{Ca}{ae}$	cos. $(\alpha - \gamma)p$
74	$+ \frac{3}{2}n^2 \cdot 2BC + \frac{bc}{\beta\gamma e^2} - \frac{Bc}{\gamma e} - \frac{Cb}{\beta e}$	cos. $(\beta + \gamma)p$
75	$+ \frac{3}{2}n^2 \cdot 2BC + \frac{bc}{\beta\gamma e^2} - \frac{Bc}{\gamma e} - \frac{Cb}{\beta e}$	cos. $(\beta - \gamma)p$
76	$\frac{3}{2}n^2 [- \frac{3A\varepsilon}{2} - \frac{An\varepsilon}{a}] + 2$	cos. $(n + \alpha)p$
77	$\frac{3}{2}n^2 [- \frac{3A\varepsilon}{2} + \frac{An\varepsilon}{a}] + 6$	cos. $(n - \alpha)p$
78	$\frac{3}{2}n^2 [- \frac{3B\varepsilon}{2} - \frac{Bn\varepsilon}{\beta}]$	cos. $(n + \beta)p$
79	$\frac{3}{2}n^2 [- \frac{3B\varepsilon}{2} + \frac{Bn\varepsilon}{\beta}]$	cos. $(n - \beta)p$
80	$\frac{3}{2}n^2 [- \frac{3C\varepsilon}{2} - \frac{Cn\varepsilon}{\gamma}]$	cos. $(n + \gamma)p$
81	$\frac{3}{2}n^2 [- \frac{3C\varepsilon}{2} + \frac{Cn\varepsilon}{\gamma}]$	cos. $(n - \gamma)p$
82	$-\frac{Bba}{2\beta a e^2} - \frac{Ab^2}{2\beta^2 e^2}$	cos. $(\alpha + 2\beta)p$
83	$-\frac{Bba}{2\beta a e^2} - \frac{Ab^2}{2\beta^2 e^2}$	cos. $(\alpha - 2\beta)p$
84	$-\frac{Cca}{2\gamma a e^2} - \frac{Ac^2}{2\gamma^2 e^2}$	cos. $(\alpha + 2\gamma)p$
85	$-\frac{Cca}{2\gamma a e^2} - \frac{Ac^2}{2\gamma^2 e^2}$	cos. $(\alpha - 2\gamma)p$
86	$-\frac{Aab}{2a\beta e^2} - \frac{Ba^2}{2a^2 e^2}$	cos. $(\beta + 2\alpha)p$
87	$-\frac{Aab}{2a\beta e^2} - \frac{Ba^2}{2a^2 e^2}$	cos. $(\beta - 2\alpha)p$

I. Æquatio. $0 =$

Ordo terminorum									
88	$-\frac{Ccb}{2\gamma\beta e^2}$	$-\frac{Bc^2}{2\gamma^2 e^2}$	.	.	.	.	.	.	$\cos. (\beta + 2\gamma) p$
89	$-\frac{Ccb}{2\gamma\beta e^2}$	$-\frac{Bc^2}{2\gamma^2 e^2}$	.	.	.	.	.	.	$\cos. (\beta - 2\gamma) p$
90	$-\frac{Aac}{2a\gamma e^2}$	$-\frac{Ca^2}{2a^2 e^2}$	.	.	.	.	.	.	$\cos. (\gamma + 2\alpha) p$
91	$-\frac{Aac}{2a\gamma e^2}$	$-\frac{Ca^2}{2a^2 e^2}$	.	.	.	.	.	.	$\cos. (\gamma - 2\alpha) p$
92	$-\frac{Bbc}{2\beta\gamma e^2}$	$-\frac{Cb^2}{2\beta^2 e^2}$	.	.	.	.	.	.	$\cos. (\gamma + 2\beta) p$
93	$-\frac{Bbc}{2\beta\gamma e^2}$	$-\frac{Cb^2}{2\beta^2 e^2}$	.	.	.	.	.	.	$\cos. (\gamma - 2\beta) p$
94	$-\frac{Abc}{2\beta\gamma e^2}$	$-\frac{Bac}{2a\gamma e^2}$	$-\frac{Cab}{2a\beta e^2}$	.	.	.	.	.	$\cos. (\alpha + \beta + \gamma) p$
95	$-\frac{Abc}{2\beta\gamma e^2}$	$-\frac{Bac}{2a\gamma e^2}$	$-\frac{Cab}{2a\beta e^2}$	.	.	.	.	.	$\cos. (\alpha + \beta - \gamma) p$
96	$-\frac{Abc}{2\beta\gamma e^2}$	$-\frac{Bac}{2a\gamma e^2}$	$-\frac{Cab}{2a\beta e^2}$	.	.	.	.	.	$\cos. (\alpha - \beta + \gamma) p$
97	$-\frac{Abc}{2\beta\gamma e^2}$	$-\frac{Bac}{2a\gamma e^2}$	$-\frac{Cab}{2a\beta e^2}$	.	.	.	.	.	$\cos. (\alpha - \beta - \gamma) p$
98	$-\frac{3}{2}n^2\varepsilon + 6$	^{VII}	.	.	.	.	.	.	$\cos. np$
99	$+\frac{9}{4}n^2\varepsilon^2$	.	.	.	.	.	.	.	$\cos. 2np$
100	$+\frac{3}{2}n^2A^2 + \frac{a^2}{2a^2e^2}$	$-\frac{Aa}{ae}$	-186	.	.	.	.	.	$\cos. 2ap$
101	$+\frac{3}{2}n^2B^2 + \frac{b^2}{2\beta^2e^2}$	$-\frac{Bb}{\beta e}$	.	.	.	.	.	.	$\cos. 2\beta p$
102	$+\frac{3}{2}n^2C^2 + \frac{c^2}{2\gamma^2e^2}$	$-\frac{Cc}{\gamma e}$	.	.	.	.	.	.	$\cos. 2\gamma p$
103	$+\frac{5}{4}n^2A^3 - \frac{Aa^2}{4a^2e^2}$	.	.	.	.	.	.	.	$\cos. 3ap$
104	$-\frac{VII}{II}$	.	.	.	.	.	.	.	$\cos. 4rp$
105	$-\frac{VII}{2I}$	.	.	.	.	.	.	.	$\cos. (4r - 2\alpha) p$
106	$-\frac{VII}{60130\frac{g}{e^2}}$	.	.	.	.	.	.	.	$\cos. 2ip$
107	-155	.	.	.	.	.	.	.	$\cos. 4ip$
108	-167	.	.	.	.	.	.	.	$\cos. (2r + 2i) p$
109	$+3600$	.	.	.	.	.	.	.	$\cos. (2r - 2i) p$
110	$+83$	.	.	.	.	.	.	.	$\cos. (2r - 4i) p$
111	-50	.	.	.	.	.	.	.	$\cos. (4r - 2i) p$

II. Æquatio. 0 =

Ordo
termi-
norum.

1	— a	fin. αp
2	— b	fin. βp
3	— c	fin. γp
4	$e^{2\frac{1}{2}t} (1 - 2\frac{1}{2}\epsilon^2) \cdot \frac{3}{2}n^2$	fin. $2rp$
5	$\frac{3}{2}n^2 \left[+\dot{a}(1 + \frac{3}{2}\epsilon^2)e^{2\frac{1}{2}t} + (2A + \frac{15A^3}{2})(1 - 2\frac{1}{2}\epsilon^2)\frac{t}{e} + A\dot{a}^2 - \frac{3A^3n}{2a} - \frac{5A^2\dot{a}}{2} \right]$	fin. $(2r + \alpha)p$
6	$\frac{3}{2}n^2 \left[-\dot{a}(1 + \frac{3}{2}\epsilon^2)e^{2\frac{1}{2}t} + (2A + \frac{15A^3}{2})(1 - 2\frac{1}{2}\epsilon^2)\frac{t}{e} + A\dot{a}^2 + \frac{3A^3n}{2a} + \frac{5A^2\dot{a}}{2} \right]$	fin. $(2r - \alpha)p$
7	$\frac{3}{2}n^2 \left[+\dot{b}(1 + \frac{3}{2}\epsilon^2)e^{2\frac{1}{2}t} + 2B(1 - 2\frac{1}{2}\epsilon^2)\frac{t}{e} \right]$. . .	fin. $(2r + \beta)p$
8	$\frac{3}{2}n^2 \left[-\dot{b}(1 + \frac{3}{2}\epsilon^2)e^{2\frac{1}{2}t} + 2B(1 - 2\frac{1}{2}\epsilon^2)\frac{t}{e} \right]$. . .	fin. $(2r - \beta)p$
9	$\frac{3}{2}n^2 \left[+\dot{c}(1 + \frac{3}{2}\epsilon^2)e^{2\frac{1}{2}t} + 2C(1 - 2\frac{1}{2}\epsilon^2)\frac{t}{e} \right]$. . .	fin. $(2r + \gamma)p$
10	$\frac{3}{2}n^2 \left[-\dot{c}(1 + \frac{3}{2}\epsilon^2)e^{2\frac{1}{2}t} + 2C(1 - 2\frac{1}{2}\epsilon^2)\frac{t}{e} \right]$. . .	fin. $(2r - \gamma)p$
11	$\frac{3}{2}n^2 \left[+2\epsilon(1 + \frac{3}{2}\epsilon^2)e^{2\frac{1}{2}t} - \frac{3}{2}\epsilon(1 - 4\epsilon^2)te^{2\frac{1}{2}t} \right]$. . .	fin. $(2r + n)p$
12	$\frac{3}{2}n^2 \left[-2\epsilon(1 + \frac{3}{2}\epsilon^2)e^{2\frac{1}{2}t} - \frac{3}{2}\epsilon(1 - 4\epsilon^2)te^{2\frac{1}{2}t} \right]$. . .	fin. $(2r - n)p$
13	$\frac{3}{2}n^2 \left[+\frac{5A^2}{2} + 2A\dot{a} + \frac{\dot{a}^2}{2} - \frac{3A^2n}{4a} \right]$	fin. $(2r + 2\alpha)p$
14	$\frac{3}{2}n^2 \left[+\frac{5A^2}{2} - 2A\dot{a} + \frac{\dot{a}^2}{2} + \frac{3A^2n}{4a} \right]$	fin. $(2r - 2\alpha)p$
15	$\frac{3}{2}n^2 \left[+\frac{5B^2}{2} + 2B\dot{b} + \frac{\dot{b}^2}{2} - \frac{3B^2n}{4\beta} \right]$	fin. $(2r + 2\beta)p$
16	$\frac{3}{2}n^2 \left[+\frac{5B^2}{2} - 2B\dot{b} + \frac{\dot{b}^2}{2} + \frac{3B^2n}{4\beta} \right]$	fin. $(2r - 2\beta)p$
17	$\frac{3}{2}n^2 \left[+\frac{5C^2}{2} + 2C\dot{c} + \frac{\dot{c}^2}{2} - \frac{3C^2n}{4\gamma} \right]$	fin. $(2r + 2\gamma)p$
18	$\frac{3}{2}n^2 \left[+\frac{5C^2}{2} - 2C\dot{c} + \frac{\dot{c}^2}{2} + \frac{3C^2n}{4\gamma} \right]$	fin. $(2r - 2\gamma)p$
19	0	fin. $(2r + 2n)p$
20	$\frac{3}{2}n^2 \left[+\frac{17\epsilon^2}{2} \right]$	fin. $(2r - 2n)p$
21	$\frac{3}{2}n^2 \left[+A\epsilon + \frac{An\epsilon}{2a} + \frac{\dot{a}\epsilon}{2} \right]$	fin. $(2r + n + \alpha)p$
22	$\frac{3}{2}n^2 \left[+A\epsilon - \frac{An\epsilon}{2a} - \frac{\dot{a}\epsilon}{2} \right]$	fin. $(2r + n - \alpha)p$

II. Aequatio. 0 =

Ordo
termi-
norum.

23	$\frac{3}{2}n^2 \left[-7A\varepsilon + \frac{7An\varepsilon}{2a} - \frac{7a\varepsilon}{2} \right]$	fin. $(2r - n + \alpha)p$
24	$\frac{3}{2}n^2 \left[-7A\varepsilon - \frac{7An\varepsilon}{2a} + \frac{7a\varepsilon}{2} \right]$	fin. $(2r - n - \alpha)p$
25	$\frac{3}{2}n^2 \left[+B\varepsilon + \frac{Bn\varepsilon}{2\beta} + \frac{b\varepsilon}{2} \right]$	fin. $(2r + n + \beta)p$
26	$\frac{3}{2}n^2 \left[+B\varepsilon - \frac{Bn\varepsilon}{2\beta} - \frac{b\varepsilon}{2} \right]$	fin. $(2r + n - \beta)p$
27	$\frac{3}{2}n^2 \left[-7B\varepsilon + \frac{7Bn\varepsilon}{2\beta} - \frac{7b\varepsilon}{2} \right]$	fin. $(2r - n + \beta)p$
28	$\frac{3}{2}n^2 \left[-7B\varepsilon - \frac{7Bn\varepsilon}{2\beta} + \frac{7b\varepsilon}{2} \right]$	fin. $(2r - n - \beta)p$
29	$\frac{3}{2}n^2 \left[+C\varepsilon + \frac{Cn\varepsilon}{2\gamma} + \frac{c\varepsilon}{2} \right]$	fin. $(2r + n + \gamma)p$
30	$\frac{3}{2}n^2 \left[+C\varepsilon - \frac{Cn\varepsilon}{2\gamma} - \frac{c\varepsilon}{2} \right]$	fin. $(2r + n - \gamma)p$
31	$\frac{3}{2}n^2 \left[-7C\varepsilon + \frac{7Cn\varepsilon}{2\gamma} - \frac{7c\varepsilon}{2} \right]$	fin. $(2r - n + \gamma)p$
32	$\frac{3}{2}n^2 \left[-7C\varepsilon - \frac{7Cn\varepsilon}{2\gamma} + \frac{7c\varepsilon}{2} \right]$	fin. $(2r - n - \gamma)p$
33	$\frac{3}{2}n^2 \left[+5AB + 2A\dot{b} + \dot{a}\dot{b} + 2B\dot{a} - \frac{3ABn}{a+\beta} \right]$	fin. $(2r + \alpha + \beta)p$
34	$\frac{3}{2}n^2 \left[+5AB - 2A\dot{b} - \dot{a}\dot{b} + 2B\dot{a} - \frac{3ABn}{a-\beta} \right]$	fin. $(2r + \alpha - \beta)p$
35	$\frac{3}{2}n^2 \left[+5AB + 2A\dot{b} - \dot{a}\dot{b} - 2B\dot{a} + \frac{3ABn}{a-\beta} \right]$	fin. $(2r - \alpha + \beta)p$
36	$\frac{3}{2}n^2 \left[+5AB - 2A\dot{b} + \dot{a}\dot{b} - 2B\dot{a} + \frac{3ABn}{a+\beta} \right]$	fin. $(2r - \alpha - \beta)p$
37	$\frac{3}{2}n^2 \left[+5AC + 2A\dot{c} + \dot{a}\dot{c} + 2C\dot{a} - \frac{3ACn}{a+\gamma} \right]$	fin. $(2r + \alpha + \gamma)p$
38	$\frac{3}{2}n^2 \left[+5AC - 2A\dot{c} - \dot{a}\dot{c} + 2C\dot{a} - \frac{3ACn}{a-\gamma} \right]$	fin. $(2r + \alpha - \gamma)p$
39	$\frac{3}{2}n^2 \left[+5AC + 2A\dot{c} - \dot{a}\dot{c} - 2C\dot{a} + \frac{3ACn}{a-\gamma} \right]$	fin. $(2r - \alpha + \gamma)p$
40	$\frac{3}{2}n^2 \left[+5AC - 2A\dot{c} + \dot{a}\dot{c} - 2C\dot{a} + \frac{3ACn}{a+\gamma} \right]$	fin. $(2r - \alpha - \gamma)p$
41	$\frac{3}{2}n^2 \left[+5BC + 2B\dot{c} + \dot{b}\dot{c} + 2C\dot{b} - \frac{3BCn}{\beta+\gamma} \right]$	fin. $(2r + \beta + \gamma)p$
42	$\frac{3}{2}n^2 \left[+5BC - 2B\dot{c} - \dot{b}\dot{c} + 2C\dot{b} - \frac{3BCn}{\beta-\gamma} \right]$	fin. $(2r + \beta - \gamma)p$
43	$\frac{3}{2}n^2 \left[+5BC + 2B\dot{c} - \dot{b}\dot{c} - 2C\dot{b} + \frac{3BCn}{\beta-\gamma} \right]$	fin. $(2r - \beta + \gamma)p$
44	$\frac{3}{2}n^2 \left[+5BC - 2B\dot{c} + \dot{b}\dot{c} - 2C\dot{b} + \frac{3BCn}{\beta+\gamma} \right]$	fin. $(2r - \beta - \gamma)p$
45	$\frac{3}{2}n^2 \left[+\frac{5A^3}{2} - \frac{11A^3n}{6a} + A\dot{a}^2 + \frac{5A^2\dot{a}}{2} \right]$	fin. $(2r + 3\alpha)p$
46	$\frac{3}{2}n^2 \left[+\frac{5A^3}{2} + \frac{11A^3n}{6a} + A\dot{a}^2 - \frac{5A^2\dot{a}}{2} \right]$	fin. $(2r - 3\alpha)p$

II. Æquatio. 0 =

Ordo
terminorum.

47	$+\frac{3\pi n^2}{8}[1+3\varepsilon^2]e^4$	fin. r
48	$\frac{3\pi n^2}{8}\left[+\frac{5A}{2}+\frac{1}{2}\dot{a}\right]$	fin. $(r+\alpha)p$
49	$\frac{3\pi n^2}{8}\left[+\frac{5A}{2}-\frac{1}{2}\dot{a}\right]$	fin. $(r-\alpha)p$
50	$\frac{3\pi n^2}{8}\left[+\frac{5B}{2}+\frac{1}{2}\dot{b}\right]$	fin. $(r+\beta)p$
51	$\frac{3\pi n^2}{8}\left[+\frac{5B}{2}-\frac{1}{2}\dot{b}\right]$	fin. $(r-\beta)p$
52	$\frac{3\pi n^2}{8}\left[+\frac{5C}{2}+\frac{1}{2}\dot{c}\right]$	fin. $(r+\gamma)p$
53	$\frac{3\pi n^2}{8}\left[+\frac{5C}{2}-\frac{1}{2}\dot{c}\right]$	fin. $(r-\gamma)p$
54	$\frac{3\pi n^2}{8}[-\varepsilon]$	fin. $(r+n)p$
55	$\frac{3\pi n^2}{8}[-3\varepsilon]$	fin. $(r-n)p$
56	$\frac{3\pi n^2}{8}\left[+\frac{15A^2}{4}-\frac{3A^2n}{8}+\frac{5A\dot{a}}{4}\right]$	fin. $(r+2\alpha)p$
57	$\frac{3\pi n^2}{8}\left[+\frac{15A^2}{4}+\frac{3A^2n}{8}-\frac{5A\dot{a}}{4}\right]$	fin. $(r-2\alpha)p$
58	$+\frac{15\pi n^2}{8}[1+3\varepsilon^2]e^4$	fin. $3r$
59	$\frac{15\pi n^2}{8}\left[+\frac{5A}{2}+\frac{3}{2}\dot{a}\right]$	fin. $(3r+\alpha)p$
60	$\frac{15\pi n^2}{8}\left[+\frac{5A}{2}-\frac{3}{2}\dot{a}\right]$	fin. $(3r-\alpha)p$
61	$\frac{15\pi n^2}{8}\left[+\frac{5B}{2}+\frac{3}{2}\dot{b}\right]$	fin. $(3r+\beta)p$
62	$\frac{15\pi n^2}{8}\left[+\frac{5B}{2}-\frac{3}{2}\dot{b}\right]$	fin. $(3r-\beta)p$
63	$\frac{15\pi n^2}{8}\left[+\frac{5C}{2}+\frac{3}{2}\dot{c}\right]$	fin. $(3r+\gamma)p$
64	$\frac{15\pi n^2}{8}\left[+\frac{5C}{2}-\frac{3}{2}\dot{c}\right]$	fin. $(3r-\gamma)p$
65	$\frac{15\pi n^2}{8}[+\varepsilon]$	fin. $(3r+n)p$
66	$\frac{15\pi n^2}{8}[-5\varepsilon]$	fin. $(3r-n)p$
67	$\frac{15\pi n^2}{8}\left[+\frac{15A^2}{4}-\frac{9A^2n}{8}+\frac{15A\dot{a}}{4}\right]$	fin. $(3r+2\alpha)p$
68	$\frac{15\pi n^2}{8}\left[+\frac{15A^2}{4}+\frac{9A^2n}{8}-\frac{15A\dot{a}}{4}\right]$	fin. $(3r-2\alpha)p$

Expositio

Expositio quantitatum cognitarum in his æquationibus.

§ 37. In his transformatis æquationibus pro cognitis habere queunt quantitates ε , $\bar{n}$, n , r , π , et i , earumque valor ex observationibus est determinandus. Est autem, ut ex superioribus rem breviter repetamus,

ε excentricitas orbitæ solaris, (§ 26.)

deinde posito motu medio Lunæ respectu fixarum $= 1$, est

$\bar{n}$ motus medius Solis, (§ 7.)

$n^2 = \frac{\odot \bar{n}^2}{\odot + \oplus}$, (§ 8.) denotante $\odot$ massam Solis, $\oplus$ massam Terræ,

$r = 1 - \bar{n}$, (§ 31.)

i motus Lunæ à nodo medius, (§ 35.)

præterea cum supra (§ 3.) posuerimus distantiam mediam Lunæ à Terra $= a$, et distantiam Solis à Terra mediam $= \frac{a}{\pi}$, (§ 17.) erit $\pi : 1$ ratio parallaxeos horizontalis Solis ad parallaxin horizontalem Lunæ mediam, ponemus autem interim illam $10''.8$, hac existente $57'.8''$, utramque vero postmodum accuratius definiemus.

§ 38. Præter hæc notandum, ex serie pro x adsumta

$$1 - A \cos. \alpha p - B \cos. \beta p - C \cos. \gamma p - \dots$$

coëfficientium unum, v. gr. A , per has æquationes non determinari, quod et natura problematis postulat; quandoquidem illud supponit excentricitatem orbitæ lunaris, utpote ab actionibus virium non pendente, esse cognitam. Itaque literæ A in sequentibus hoc munus, nisi expressis verbis aliud indicemus, demandabimus, ut excentricitatem orbitæ Lunæ designet; quo pacto vero litera α motum anomalæ mediæ, sive $1 - \alpha$ motum apogæi lunaris repræsentare debet, ejusque valor ex termino primo æquationis I. erit determinandus.

§ 39. Spectari ergo debet hæc litera α tanquam incognita aut indeterminata, saltem in hoc primo termino; in reliquis tamen, ubi ea cum aliis r , n , i , combinatur, licebit ejus valorem verum ex observationibus supponere, eo fine, ut tanto facilius exactiorque evadat determinatio reliquarum quantitatum. Quodsi vero absoluto hoc determinandi negotio ex primo termino æquationis I. idem valor pro litera α prodeat, quem observationes ostendunt; indicio id erit, theoriæ gravitatis Newtonianæ, qua vires Solis, Terræ, atque Lunæ, quadratis distantiarum reciproce statuuntur proportionales, (§ 6.) veritati atque observationibus esse consentaneam; sin minus hoc accidat, calculo ceteroquin accurate peracto, correctione eam indigere docebit.

§ 40. Progrediendo igitur ad numeros, literarum harum hos obtinebimus valores

$\bar{n} = 0,0748017$	$l\bar{n} = 8.873911$
$\alpha = 0,9915478$	$l\alpha = 9.996314$
$r = 0,9251983$	$lr = 9.966235$
$i = 1,0040194$	$li = 0.001742$
$n^2 = \bar{n}^2 = 0,0055953$ *	$ln^2 = 7.747822$
$\pi = 0,0031505$	$l\pi = 7.498380$
$A = 0,0545400$	$lA = 8.736715$
$\varepsilon = 0,0168350$	$l\varepsilon = 8.226213$

§ 41. In æquationibus nostris (§ 36.) occurrunt adhuc quantitates e et t tanquam coëfficientes, quarum valores à literis A , B , C , a , b , c , &c. pendere ex superioribus (§ 21. 31, 32.) manifestum est. Utraque vero ab unitate haud multum discrepare potest, cum valores literarum A , B , C , a , b , c , &c. quorum quadrata tantum in e et t

* Cum massa Solis sit ad massam Terræ ut 220000 ad 1. circiter, patet summo jure poni posse $n^2 = \bar{n}^2 = \frac{\odot n^2}{\odot + \oplus}$.

ingrediuntur, jam in se sint satis exigui, præter A; sed ex quo hic nullum dubium oriri potest, quia A inter cognitæ refertur, (§ 38.) Quodsi igitur literarum reliquarum B, C, a, b, c, &c. valores tantum proxime veri undecunque noti fuerint, satis accurate inde defini poterunt literæ e et t. Quamobrem ex calculis meis prioribus hos quoque valores cognitos adsumam statuamque

$$e = 1,0047450 \quad l e = 0.002056$$

$$t = 0,9999183 \quad l t = 9.999964$$

quos quidem multo exactiores esse, quam hoc loco requirantur, absoluto hoc calculo patebit.

§ 42. Denique cum in secundo et tertio termino æquationis I. coëfficientes literarum B, C, obveniant hi

$$\beta^2 = 1 + \frac{3n^2}{2e^2} \left(1 + \frac{3}{2}\varepsilon^2\right) - \frac{3b^2}{4\beta^2 e^2} - \frac{a^2}{2a^2 e^2} - \frac{c^2}{2\gamma^2 e^2},$$

$$\text{et } \gamma^2 = 1 + \frac{3n^2}{2e^2} \left(1 + \frac{3}{2}\varepsilon^2\right) - \frac{3c^2}{4\gamma^2 e^2} - \frac{a^2}{2a^2 e^2} - \frac{b^2}{2\beta^2 e^2},$$

quorum usus in determinandis literis B, C, ... frequentissimus est, hos quoque, quantum licet, in numeris exprimere conveniens erit. Quem in finem statuamus brevitatis causa

$$1 - \frac{3n^2}{2e^2} \left(1 + \frac{3}{2}\varepsilon^2\right) - \frac{a^2}{2a^2 e^2} - \frac{b^2}{2\beta^2 e^2} - \frac{c^2}{2\gamma^2 e^2} = f^2,$$

ut isti coëfficientes sint

$$\beta^2 = f^2 - \frac{b^2}{4\beta^2 e^2},$$

$$\gamma^2 = f^2 - \frac{c^2}{4\gamma^2 e^2},$$

cum ergo dentur n, ε, et e, erit

$$f^2 = 0,9916826 - \frac{a^2}{2a^2 e^2} - \frac{b^2}{2\beta^2 e^2} - \frac{c^2}{2\gamma^2 e^2},$$

ex calculo autem meo priori inveni

$$\frac{a^2}{2a^2 e^2} + \frac{b^2}{2\beta^2 e^2} + \frac{c^2}{2\gamma^2 e^2} = 0,0000112,$$

unde erit perquam accurate

$$f^2 = 0,9916714 \text{ et } f = 0,9958270,$$

quoniam vero hoc additamentum ex terminis $\frac{a^2}{2a^2 e^2}$, ... &c. ortum tam exiguum est, ut prorsus negligi potuisset, liquet jure multo majori obmitti

obmitti posse partes $\frac{b^2}{4\beta^2e^2}$ et $\frac{c^2}{4\gamma^2e^2}$, quæ istis coefficientibus adhærent; est enim casu quo $\beta = 2r$, $\frac{b^2}{4\beta^2e^2} = 52$ tantum, et casu quo $\gamma = 2r - \alpha$, $\frac{c^2}{4\gamma^2e^2} = 3$; in reliquis casibus hæ quantitates prorsus evanescunt. Simpliciter ergo tribuemus coefficientibus his hanc formam

$$\beta^2 - f^2 \text{ five } (\beta + f)(\beta - f),$$

$$\gamma^2 - f^2 \text{ . . } (\gamma + f)(\gamma - f),$$

hoc unico observato, quod casu quo $\beta = 2r$, f fit $= 0,9958244$, in omnibus reliquis autem $f = 0,9958270$.

§ 43. Designante jam α motum anomalix mediæ Lunæ,

Si β vel $\gamma =$ erit $l\beta$ vel $\gamma =$ et $l(\beta^2 - f^2)$ vel $l(\gamma^2 - f^2) =$

$2r$	0.267265 +	0.386018 +
$2r + \alpha$	0.453615 +	0.850338 +
$2r - \alpha$	9.933916 +	9.404916 —
$4r$	0.568295 +	1.103947 +
$4r + \alpha$	0.671380 +	1.322746 +
$4r - \alpha$	0.432848 +	0.802660 +
$2r + n$	0.284475 +	0.433723 +
$2r - n$	0.249344 +	0.334637 +
$2r + 2\alpha$	0.583580 +	1.136831 +
$2r - 2\alpha$	9.122868 —	9.988587 —
$2r - 2n$	0.230652 +	0.278987 +
$2r + n + \alpha$	0.464898 +	0.875971 +
$2r + n - \alpha$	9.970184 +	9.079064 —
$2r - n + \alpha$	0.442031 +	0.823827 +
$2r - n - \alpha$	9.894342 +	9.576274 —

Si

Si $\gamma =$ crit $\gamma =$ et $l(\gamma^2 - f^2) =$

$\#$	8.873911 +	9.993910 —
$\alpha + n$	0.027899 +	9.162654 +
$\alpha - n$	9.962249 +	9.179690 —
2α	0.297344 +	0.468494 +
$4r - 2\alpha$	0.234946 +	0.291993 +
$2n$	9.174941 +	9.986410 —
$2r - 3\alpha$	0.050861 —	9.434982 +
$4r - 3\alpha$	9.861030 +	9.666820 —
r	9.966235 +	9.132513 —
$r + \alpha$	0.282550 +	0.428500 +
$r - \alpha$	8.821838 —	9.994434 —
$r + n$	0.000000 +	7.920571 +
$r - n$	9.929622 —	9.428900 —
$3r$	0.443356 +	0.826868 +
$2i$	0.302772 +	0.482951 +
$2r + 2i$	0.58641 +	1.14288 +
$2r - 2i$	9.197672 —	9.985346 —
$2i + \alpha$	0.47706 +	0.90341 +
$2i - \alpha$	0.007103 +	8.618910 +
$2i + n$	0.31866 +	0.52460 +
$2i - n$	0.28628 +	0.43865 +
$2r - 2i + \alpha$	9.92111 +	9.47162 —
$2r - 2i - \alpha$	0.06039 —	9.51724 +
$2r - 2\alpha + 2i$	0.27308 +	0.40230 +
$2r - 2\alpha - 2i$	0.33056 —	0.55661 +
$2r - 2i + n$	8.918243 —	9.99334 —
$2r - 2i - n$	9.366317 —	9.97201 —
$2\alpha - 2i$	8.396952 —	9.99608 —

Signa + vel — quæ logarithmis subjunxi, non ad ipsos logarithmos spectant, quippe qui omnes positivi sunt habendi, sed ad numeros, quorum illi sunt logarithmi.

§ 44. Ad caculum juxta æquationes nostras principales reductas (§ 36.) eo facilius instituendum, notandi adhuc sunt quantitatū in illis obviarum hi valores logarithmici, ex § 40 et 41. derivati,

$$\begin{array}{ll}
 l \frac{3}{2} n^2 \dots\dots\dots = 7.923913 & l e^2 \dots\dots\dots = 0.004112 \\
 l \frac{3}{2} n^2 (1 + \frac{3}{2} \epsilon^2) \dots\dots = 7.924098 & l e^{2\frac{1}{2}} \dots\dots\dots = 0.004797 \\
 l \frac{3}{2} n^2 (1 - 2\frac{1}{2} \epsilon^2) \frac{t}{e^2} \dots\dots = 7.919458 & l e \dots\dots\dots = 0.002056 \\
 l \frac{3}{2} n^2 (1 - 2\frac{1}{2} \epsilon^2) t \dots\dots = 7.923570 & l e^{1\frac{1}{2}} \dots\dots\dots = 0.002745 \\
 l \frac{3}{2} n^2 (1 - 4\epsilon^2) t \dots\dots = 7.923385 & l(1 + \frac{3}{2} \epsilon^2) = 0.000185 \\
 l \frac{3}{2} n^2 (1 + \frac{3}{2} \epsilon^2) e^{2\frac{1}{2}} \dots\dots = 7.928895 & l(1 + 3\epsilon^2) = 0.00370 \\
 l \frac{3}{2} n^2 (1 - 2\frac{1}{2} \epsilon^2) t e^{2\frac{1}{2}} \dots\dots = 7.928367 & l(1 - 2\frac{1}{2} \epsilon^2) = 9.999693 \\
 l \frac{3}{2} n^2 (1 - 2\frac{1}{2} \epsilon^2) \frac{t}{e} \dots\dots = 7.921514 & l(1 - 4\epsilon^2) = 9.999508 \\
 l \frac{3}{2} n^2 (1 - 4\epsilon^2) t e^{2\frac{1}{2}} \dots\dots = 7.928182 & \\
 l(1 + \frac{5A^2}{2}) \dots\dots\dots = 0.003218 & \\
 l(1 + \frac{15A^2}{4}) \dots\dots\dots = 0.004818 &
 \end{array}$$

Determinatio quantitatū in æquationibus § 36. contentarum.

§ 45. Longa res esset et tædii plenissima, nec facile, nisi quis calculos omnes ipse ponat, intelligenda, si ostenderem, qua ratione singulæ quantitates indeterminatæ A, B, C, &c. a, b, c, &c. ex æquationibus principalibus § 36. deriventur. Neque tamen, præter prolixitatem, novi quid aut insoliti contineret methodus, quandoquidem ipsa nihil differt à vulgari quantitatū indeterminatarum valores determinandi via omnibus nota, qui analysi recentiorum vel leviter imbuti sunt. Itaque cum sufficiat, peritis viam obiter monstrasse, qua ad cognoscendas Lunæ inæqualitates ipse pervenerim, quod hætenus spero ex abundanti factum, adscribam valores tantum veros quantitatū A, B, C, &c. a, b, c, &c. quantos eos adhibita summa cura et attentione, repetito etiam in pluribus locis cal-

culo terque quaterque, inde obtinui. Igitur pro serie supra § 19. adsumta

$$\frac{dP}{dp} = a \text{ fin. } \alpha p + b \text{ fin. } \beta p + c \text{ fin. } \gamma p + \dots$$

prodiit hæc determinata

$$\begin{aligned} \frac{dP}{dp} = & + 0,0001896 \text{ fin. } \alpha p \\ & + 84784 \text{ fin. } 2rp \\ & + 8502 \text{ fin. } (2r + \alpha) p \\ & + 9892 \text{ fin. } (2r - \alpha) p \\ & - 687 \text{ fin. } 4rp \\ & - 135 \text{ fin. } (4r + \alpha) p \\ & + 1660 \text{ fin. } (4r - \alpha) p \\ & + 710 \text{ fin. } (2r + n) p \\ & - 4957 \text{ fin. } (2r - n) p \\ & + 539 \text{ fin. } (2r + 2\alpha) p \\ & + 692\frac{1}{2} \text{ fin. } (2r - 2\alpha) p \\ & + 204 \text{ fin. } (2r - 2n) p \\ & + 74 \text{ fin. } (2r + n + \alpha) p \\ & + 31 \text{ fin. } (2r + n - \alpha) p \\ & - 480 \text{ fin. } (2r - n + \alpha) p \\ & - 565 \text{ fin. } (2r - n - \alpha) p \\ & + 0 \text{ fin. } np \\ & - 50 \text{ fin. } (\alpha + n) p \\ & - 124 \text{ fin. } (\alpha - n) p \\ & - 241 \text{ fin. } 2\alpha p \\ & + 397 \text{ fin. } (4r - 2\alpha) p \\ & - 44 \text{ fin. } 3\alpha p \\ & 0 \text{ fin. } 2np \\ & + 130 \text{ fin. } rp \\ & + 32 \text{ fin. } (r + \alpha) p \\ & + 17 \text{ fin. } (r - \alpha) p \\ & + 396 \text{ fin. } 3rp \\ & - \frac{1}{2} \text{ fin. } (r + n) p \\ & - 7\frac{1}{2} \text{ fin. } (r - n) p \end{aligned}$$

$$\begin{aligned}
& + 58 \text{ fin. } (2r - 3\alpha) p \\
& - 13 \text{ fin. } (4r - 3\alpha) p \\
& - 120 \text{ fin. } 2ip \\
& + 321 \text{ fin. } (2r + 2i) p \\
& + 339 \text{ fin. } (2r - 2i) p \\
& - 26 \text{ fin. } (2i + \alpha) p \\
& - 28 \text{ fin. } (2i - \alpha) p \\
& - 1 \text{ fin. } (2i + n) p \\
& + 6 \text{ fin. } (2i - n) p \\
& - 1 \text{ fin. } (2r - 2i + \alpha) p \\
& + 54 \text{ fin. } (2r - 2i - \alpha) p \\
& - 7 \text{ fin. } (2r - 2\alpha + 2i) p \\
& + 0 \text{ fin. } (2r - 2\alpha - 2i) p \\
& + 2\frac{3}{4} \text{ fin. } (2r - 2i + n) p \\
& - 21 \text{ fin. } (2r - 2i - n) p \\
& + \frac{3}{4} \text{ fin. } (2\alpha - 2i) p.
\end{aligned}$$

Altera autem series

$x = 1 - A \cos. \alpha p - B \cos. \beta p - C \cos. \gamma p - "$
 five $1 - x = A \cos. \alpha p + B \cos. \beta p + C \cos. \gamma p,$
 post peractam determinationem hanc formam induit.

$$\begin{aligned}
1 - x = & + 0,0545400 \cos. \alpha p \\
& - 70246 \cos. 2rp \\
& - 1361 \cos. (2r + \alpha) p \\
& + 110382 \cos. (2r - \alpha) p \\
& + 36 \cos. 4r \\
& + 5 \cos. (4r + \alpha) p \\
& - 331 \cos. (4r - \alpha) p \\
& - 512 \cos. (2r + n) p \\
& + 4813 \cos. (2r - n) p \\
& - 31 \cos. (2r + 2\alpha) p \\
& - 10707 \cos. (2r - 2\alpha) p \\
& - 233 \cos. (2r - 2n) p
\end{aligned}$$

$$\begin{aligned}
& - 11 \text{ cof. } (2r + n + \alpha) p \\
& + 444 \text{ cof. } (2r + n - \alpha) p \\
& + 80 \text{ cof. } (2r - n + \alpha) p \\
& - 4642 \text{ cof. } (2r - n - \alpha) p \\
& - 1075 \text{ cof. } np \\
& + 1700 \text{ cof. } (\alpha + n) p \\
& - 2790 \text{ cof. } (\alpha - n) p \\
& + 204 \text{ cof. } 2\alpha p \\
& - 339 \text{ cof. } (4r - 2\alpha) p \\
& + 36 \text{ cof. } 2np \\
& + 3780 \text{ cof. } rp \\
& - 33 \text{ cof. } (r + \alpha) p \\
& - 491 \text{ cof. } (r - \alpha) p \\
& - 94 \text{ cof. } 3rp \\
& + 573 \text{ cof. } (r + n) p \\
& - 118 \text{ cof. } (r - n) p \\
& - 689 \text{ cof. } (2r - 3\alpha) p \\
& + 7 \text{ cof. } (4r - 3\alpha) p \\
& + 20026 \text{ cof. } 2ip \\
& - 10 \text{ cof. } (2r + 2i) p \\
& - 543 \text{ cof. } (2r - 2i) p \\
& + 8 \text{ cof. } (2i + \alpha) p \\
& - 2822 \text{ cof. } (2i - \alpha) p \\
& - 46 \text{ cof. } (2i + n) p \\
& + 58 \text{ cof. } (2i - n) p \\
& + 208 \text{ cof. } (2r - 2i + \alpha) p \\
& - 771 \text{ cof. } (2r - 2\alpha - \alpha) p \\
& + 119 \text{ cof. } (2r - 2\alpha + 2i) p \\
& - 63 \text{ cof. } (2r - 2\alpha - 2i) p \\
& + 17 \text{ cof. } (2r - 2i + n) p \\
& - 43 \text{ cof. } (2r - 2i - n) p \\
& + 145 \text{ cof. } (2\alpha - 2i) p.
\end{aligned}$$

§ 46. Plures in hac utraque serie occurrunt termini, adeo exigui ut omnino negligi possint, quos tamen ideo retinui, ut adpareat, quousque adproximationem persecutus sim. Præter has autem series determinati quoque indidem sunt valores literarum g et α ; scil.

$$g = 1,018408$$

$$\text{et } \alpha = 0,9915965$$

quarum ista inservit ad distantiam Lunæ à Terra mediam invenien-
dam, hæc vero motum apogæi lunaris, quantus quidem juxta theo-
riam esse debeat, complectitur. Cum enim sit $\alpha : 1$ ratio anomalie
Lunæ ad motum longitudinis medium erit $1 - \alpha$ motus apogæi in-
consequentia; qui proinde est $0,0084035$ siquidem motus longitu-
dinis Lunæ $= 1$; quodsi vero motus medius longitudinis Lunæ di-
urnus ponatur juxta observationes $= 13^\circ. 10'. 35''$, erit motus di-
urnus apogæi Lunæ medius secundum theoriam $= 0^\circ. 6'. 38''. 37'''$,
at juxta observationes idem est $= 0^\circ. 6'. 40''. 56'''$ deficit igitur iste
à vero vix parte ducentesima, adeoque theoriam Newtonianam egre-
gie confirmat. (Vide § 39.)

§ 47. Cognitis jam literis A, B, C, a, b, c , &c. facile inde ad-
hibendo æquationes supra § 21 et 22. expositas reperitur valor tam
longitudinis veræ φ quam mediæ q per p expressus. Calculo autem
peracto erit

$$\begin{aligned} \varphi = p + & \overset{\text{VII}}{1920} \text{ fin. } \alpha p \\ & - 61 \text{ fin. } 2\alpha p \\ & - 5 \text{ fin. } 3\alpha p \\ & + 24645 \text{ fin. } 2rp \\ & - 50 \text{ fin. } 4rp \\ & + 1048 \text{ fin. } (2r + \alpha)p \\ & + 13348 \text{ fin. } (2r - \alpha)p \\ & + 134 \text{ fin. } (4r - 2\alpha)p \\ & - 6 \text{ fin. } (4r + \alpha)p \\ & + 225 \text{ fin. } (4r - \alpha)p \end{aligned}$$

L

+ 191

- + 191 fin. $(2r + n)p$
- 1564 fin. $(2r - n)p$
- + 36 fin. $(2r + 2\alpha)p$
- + 39146 fin. $(2r - 2\alpha)p$
- o fin. $(4r - 4\alpha)p$
- + 71 fin. $(2r - 2n)p$
- + 8 fin. $(2r + n + \alpha)p$
- + 35 fin. $(2r + n - \alpha)p$
- 62 fin. $(2r - n + \alpha)p$
- 915 fin. $(2r - n - \alpha)p$
- o fin. np
- o fin. $2np$
- 44 fin. $(\alpha + n)p$
- 147 fin. $(\alpha - n)p$
- + 37 fin. $(2r - 3\alpha)p$
- 26 fin. $(4r - 3\alpha)p$
- + 151 fin. rp
- + 9 fin. $(r + \alpha)p$
- + 3866 fin. $(r - \alpha)p$
- o fin. $(r + n)p$
- 10 fin. $(r - n)p$
- + 51 fin. $3rp$
- 31 fin. $2ip$
- + 22 fin. $(2r + 2i)p$
- + 13577 fin. $(2r - 2i)p$
- 3 fin. $(2i + \alpha)p$
- 27 fin. $(2i - \alpha)p$
- o fin. $(2i + n)p$
- + 2 fin. $(2i - n)p$
- 1 fin. $(2r - 2i + \alpha)p$
- + 41 fin. $(2r - 2i - \alpha)p$
- 2 fin. $(2r - 2\alpha + 2i)p$
- o fin. $(2r - 2\alpha - 2i)p$

$$\begin{aligned}
 &+ 392 \text{ fin. } (2r - 2i + n) p \\
 &- 387 \text{ fin. } (2r - 2i - n) p \\
 &+ 1216 \text{ fin. } (2\alpha - 2i) p.
 \end{aligned}$$

Porro erit

$$\begin{aligned}
 q = p &+ 0,1097735 \text{ fin. } \alpha p \\
 &+ 22688 \text{ fin. } 2\alpha p \\
 &+ 575 \text{ fin. } 3\alpha p \\
 &- 66738 \text{ fin. } 2rp \\
 &+ 149 \text{ fin. } 4rp \\
 &- 4627 \text{ fin. } (2r + \alpha) p \\
 &+ 242875 \text{ fin. } (2r - \alpha) p \\
 &+ 590 \text{ fin. } (4r - 2\alpha) p \\
 &+ 9 \text{ fin. } (4r + \alpha) p \\
 &- 1009 \text{ fin. } (4r - \alpha) p \\
 &- 457 \text{ fin. } (2r + n) p \\
 &+ 4989 \text{ fin. } (2r - n) p \\
 &- 237 \text{ fin. } (2r + 2\alpha) p \\
 &+ 32172 \text{ fin. } (2r - 2\alpha) p \\
 &+ 143 \text{ fin. } (4r - 4\alpha) p \\
 &- 134 \text{ fin. } (2r - 2n) p \\
 &- 49 \text{ fin. } (2r + n + \alpha) p \\
 &+ 889 \text{ fin. } (2r + n - \alpha) p \\
 &+ 363 \text{ fin. } (2r - n + \alpha) p \\
 &- 10989 \text{ fin. } (2r - n - \alpha) p \\
 &- 33984 \text{ fin. } np \\
 &+ 504 \text{ fin. } 2np \\
 &+ 3094 \text{ fin. } (\alpha + n) p \\
 &- 6157 \text{ fin. } (\alpha - n) p \\
 &+ 1902 \text{ fin. } (2r - 3\alpha) p \\
 &- 218 \text{ fin. } (4r - 3\alpha) p \\
 &+ 8048 \text{ fin. } rp \\
 &+ 286 \text{ fin. } (r + \alpha) p \\
 &+ 3810 \text{ fin. } (r - \alpha) p
 \end{aligned}$$

+ 1155

$$\begin{aligned}
&+ 1155 \text{ fin. } (r + n) p \\
&- 293 \text{ fin. } (r - n) p \\
&- 97 \text{ fin. } 3rp \\
&+ 19812 \text{ fin. } 2ip \\
&- 94 \text{ fin. } (2r + 2i) p \\
&+ 9673 \text{ fin. } (2r - 2i) p \\
&+ 1092 \text{ fin. } (2i + \alpha) p \\
&- 2391 \text{ fin. } (2i - \alpha) p \\
&- 46 \text{ fin. } (2i + n) p \\
&+ 60 \text{ fin. } (2i - n) p \\
&+ 402 \text{ fin. } (2r - 2i + \alpha) p \\
&+ 904 \text{ fin. } (2r - 2i - \alpha) p \\
&+ 91 \text{ fin. } (2r - 2\alpha + 2i) p \\
&+ 148 \text{ fin. } (2r - 2\alpha - 2i) p \\
&- 369 \text{ fin. } (2r - 2i + n) p \\
&+ 218 \text{ fin. } (2r - 2i - n) p \\
&- 579 \text{ fin. } (2\alpha - 2i) p.
\end{aligned}$$

Reductio harum formularum ad formam commodiorem.

§ 48. Formulæ hæ, uti adparet, ita comparatæ sunt, ut dato ad quodvis tempus propositum angulo p inveniri inde queat longitudo Lunæ tam vera φ quam media q . Usus igitur illarum ad calculum astronomicum minus aptus est, quippe in hoc longitudo vera ex media quæritur. Verum haud difficile est, ex iisdem formulis eliminando angulum p æquationem inter φ et q derivare, quæ magis ad rem faciat. Primum enim ex æquatione posteriore, adhibita methodo revertendi series, invenietur

$$\begin{aligned}
p &= q - 0,1096765 \text{ fin. } \alpha q \\
&+ 37447 \text{ fin. } 2\alpha q \\
&- 1734 \text{ fin. } 3\alpha q \\
&+ 91152 \text{ fin. } 2r q
\end{aligned}$$

+	456	fin. $4rq$
—	7922	fin. $(2r + \alpha)q$
—	237637	fin. $(2r - \alpha)q$
+	1899	fin. $(4r - 2\alpha)q$
—	15	fin. $(4r + \alpha)q$
—	1669	fin. $(4r - \alpha)q$
+	824	fin. $(2r + n)q$
—	6381	fin. $(2r - n)q$
+	247	fin. $(2r + 2\alpha)q$
—	30423	fin. $(2r - 2\alpha)q$
—	153	fin. $(4r - 4\alpha)q$
+	133	fin. $(2r - 2n)q$
—	36	fin. $(2r + n + \alpha)q$
—	1262	fin. $(2r + n - \alpha)q$
+	527	fin. $(2r - n + \alpha)q$
+	11075	fin. $(2r - n - \alpha)q$
+	33983	fin. nq
—	497	fin. $2nq$
—	5109	fin. $(\alpha + n)q$
+	7799	fin. $(\alpha - n)q$
—	49	fin. $(2r - 3\alpha)q$
+	519	fin. $(4r - 3\alpha)q$
—	7867	fin. rq
+	544	fin. $(r + \alpha)q$
—	3774	fin. $(r - \alpha)q$
—	1166	fin. $(r + n)q$
+	305	fin. $(r - n)q$
+	20	fin. $3rq$
—	19912	fin. $2iq$
—	310	fin. $(2r + 2i)q$
—	9688	fin. $(2r - 2i)q$
+	2203	fin. $(2i + \alpha)q$
+	1434	fin. $(2i - \alpha)q$

$$\begin{aligned}
& - 26 \text{ fin. } (2i + n) q \\
& + 3 \text{ fin. } (2i - n) q \\
& + 34 \text{ fin. } (2r - 2i + \alpha) q \\
& - 20 \text{ fin. } (2r - 2i - \alpha) q \\
& - 40 \text{ fin. } (2r - 2\alpha + 2i) q \\
& - 86 \text{ fin. } (2r - 2\alpha - 2i) q \\
& + 369 \text{ fin. } (2r - 2i + n) q \\
& - 218 \text{ fin. } (2r - 2i - n) q \\
& + 579 \text{ fin. } (2\alpha - 2i) q.
\end{aligned}$$

Deinde substituto hoc valore ipsius p in singulis terminis æquationis prioris, obtinebitur

$$\begin{aligned}
\varphi = q & - 0,1094159 \text{ fin. } \alpha q \\
& + 37308 \text{ fin. } 2\alpha q \\
& - 1732 \text{ fin. } 3\alpha q \\
& + 115046 \text{ fin. } 2rq \\
& + 600 \text{ fin. } 4rq \\
& - 9374 \text{ fin. } (2r + \alpha) q \\
& - 221514 \text{ fin. } (2r - \alpha) q \\
& + 1779 \text{ fin. } (4r - 2\alpha) q \\
& - 28 \text{ fin. } (4r + \alpha) q \\
& - 1925 \text{ fin. } (4r - \alpha) q \\
& + 1092 \text{ fin. } (2r + n) q \\
& - 7969 \text{ fin. } (2r - n) q \\
& + 333 \text{ fin. } (2r + 2\alpha) q \\
& + 9368 \text{ fin. } (2r - 2\alpha) q \\
& - 153 \text{ fin. } (4r - 4\alpha) q \\
& + 204 \text{ fin. } (2r - 2n) q \\
& - 43 \text{ fin. } (2r + n + \alpha) q \\
& - 1188 \text{ fin. } (2r + n - \alpha) q \\
& + 610 \text{ fin. } (2r - n + \alpha) q \\
& + 9992 \text{ fin. } (2r - n - \alpha) q \\
& + 33983 \text{ fin. } nq
\end{aligned}$$

—	497	fin. $2nq$
—	5171	fin. $(\alpha + n)q$
+	7615	fin. $(\alpha - n)q$
—	282	fin. $(2r - 3\alpha)q$
+	549	fin. $(4r - 3\alpha)q$
—	7702	fin. rq
+	545	fin. $(r + \alpha)q$
+	100	fin. $(r - \alpha)q$
—	1166	fin. $(r + n)q$
+	295	fin. $(r - n)q$
+	71	fin. $3rq$
—	19931	fin. $2iq$
—	333	fin. $(2r + 2i)q$
+	3933	fin. $(2r - 2i)q$
+	2200	fin. $(2i + \alpha)q$
+	1433	fin. $(2i - \alpha)q$
—	26	fin. $(2i + n)q$
+	5	fin. $(2i - n)q$
—	84	fin. $(2n - 2i + \alpha)q$
+	138	fin. $(2r - 2i - \alpha)q$
—	33	fin. $(2r - 2\alpha + 2i)q$
—	91	fin. $(2r - 2\alpha - 2i)q$
+	761	fin. $(2r - 2i + n)q$
—	605	fin. $(2r - 2i - n)q$
+	1795	fin. $(2\alpha - 2i)q$

§ 49. Quanquam hæc formula ad longitudinem Lunæ veram ϕ ex media q inveniendam satis apta sit atque in tabulas astronomicas facile redigi possit, in quibus argumenta inæqualitatum per motus medios anomalix, distantix à Sole et nodo exprimantur, eo prorsus modo, quo suas disposuit tabulas celeb. Clairaut; ingens tamen numerus inæqualitatum, et tabularum inde construendarum calculum loci

Lunæ

Lunæ adeo operosum ac molestum redderet, ut vel patientissimo calculatori tædium parere eumque ab examine tabularum detertere posset. Idcirco viam quæfivi, qua ad pauciores redigi queant inæqualitates, quas ista formula complectitur, et quidem tractanti hanc rem universam ab initio ad finem facile adparebat, plures inæqualitates inde ortas esse, quod in calculum medius motus Solis introductus sit, quæ disparituræ erant, si verus Solis locus fuisset adhibitus; deinde terminos $-5171 \text{ fin. } (\alpha + n)q$ et $+7615 \text{ fin. } (\alpha - n)q$ maximam partem tolli adplicando anomaliam mediæ Lunæ αq , antequam per hanc æquatio centri excerpatur, æquationem peculiarem, quæ ab anomalia media Solis nq pendeat. Denique etiam alias inæqualitates vel minui, vel evanescere prorsus, si eidem anomaliam Lunæ adplicentur inæqualitates longitudinis minores, excepta parte earum, quæ à distantia Lunæ à Sole $r q$ et à nodo $i q$ pendent. His igitur omnibus ac singulis observatis si brevitatis gratia ponatur:

- p Anomalia Lunæ mediæ. —
- $\tilde{p}$ Anomalia Lunæ correctæ, hoc est ea, quæ prodit adplicatis mediæ anomaliam, inæqualitatibus minoribus, exceptis illis quæ distantiam Lunæ à Sole, aut à nodo pro argumento habent.
- s Anomalia mediæ Solis.
- ω Distantia loci Lunæ medii à loco vero Solis.
- $\tilde{\omega}$ Distantia loci Lunæ æquati à loco Solis vero.
- δ Distantia medii loci Lunæ à loco nodi correcto.
- $\tilde{\delta}$ Distantia loci Lunæ æquati, et per variationem correcti à loco nodi correcto.

Sique præterea anomaliam Lunæ mediæ adplicetur correctio

$$+ 22'. 6'' \text{ fin. } s - 15'' \text{ fin. } 2s;$$

erit reductione formulæ proxime præcedentis rite instituta

In partibus graduum.

$\varphi = q$	—	0,1096465	fin. $\dot{p}$	.	.	.	.	.	—	6. 16. 56,2
+		37669	fin. $2\dot{p}$	.	.	.	.	.	+	12. 57,0
—		1722	fin. $3\dot{p}$	.	.	.	.	.	—	0. 35,5
+		103850	fin. $2\tilde{\omega}$	.	.	.	.	.	+	35. 42,1
+		246	fin. $4\tilde{\omega}$	.	.	.	.	.	+	0. 5,0
+		2665	fin. $(2\omega + \dot{p})$	.	.	.	.	.	+	0. 55,0
—		232298	fin. $(2\omega - \dot{p})$	.	.	.	.	.	—	1. 19. 51,5
+		1857	fin. $(4\omega - 2\dot{p})$	.	.	.	.	.	+	0. 38,3
+		122	fin. $(4\omega + \dot{p})$	.	.	.	.	.	+	0. 2,5
+		441	fin. $(4\omega - \dot{p})$	.	.	.	.	.	+	0. 9,1
—		2831	fin. $(2\omega + s)$	.	.	.	.	.	—	0. 58,4
—		3534	fin. $(2\omega - s)$	.	.	.	.	.	—	1. 12,9
—		1232	fin. $(2\omega + 2\dot{p})$	.	.	.	.	.	—	0. 25,6
—		3013	fin. $(2\omega - 2\dot{p})$	.	.	.	.	.	—	1. 2,1
—		123	fin. $(4\omega - 4\dot{p})$	.	.	.	.	.	—	0. 2,5
—		26	fin. $(2\omega - 2s)$	.	.	.	.	.	—	0. 0,5
+		426	fin. $(2\omega + \dot{p} + s)$	.	.	.	.	.	+	0. 8,7
+		6341	fin. $(2\omega - \dot{p} + s)$	.	.	.	.	.	+	2. 10,8
—		305	fin. $(2\omega + \dot{p} - s)$	.	.	.	.	.	—	0. 6,3
+		2113	fin. $(2\omega - \dot{p} - s)$	.	.	.	.	.	+	0. 43,6
+		33902	fin. s	.	.	.	.	.	+	11. 39,3
—		497	fin. $2s$	.	.	.	.	.	—	0. 10,1
+		566	fin. $(\dot{p} + s)$	.	.	.	.	.	+	0. 11,7
+		1942	fin. $(\dot{p} - s)$	.	.	.	.	.	+	0. 40,0
+		1093	fin. $(2\omega - 3\dot{p})$	.	.	.	.	.	+	0. 22,5
+		585	fin. $(4\omega - 3\dot{p})$	.	.	.	.	.	+	0. 12,1
—		7691	fin. $\tilde{\omega}$	.	.	.	.	.	—	2. 38,6
+		126	fin. $(\omega + \dot{p})$	.	.	.	.	.	+	0. 2,6
+		519	fin. $(\omega - \dot{p})$	.	.	.	.	.	+	0. 10,7
—		1024	fin. $(\omega + s)$	.	.	.	.	.	—	0. 21,1
+		153	fin. $(\omega - s)$	.	.	.	.	.	+	0. 3,2
			N							+ 71

In partibus graduum.

+	71	fin. $3\tilde{\omega}$		+	0.	0.	1,5
—	19932	fin. $2\tilde{\delta}$		—		6.51,2	
—	102	fin. $(2\omega + 2\delta)$		—		0. 2,1	
+	4164	fin. $(2\omega - 2\delta)$		+		1.25,9	
+	8	fin. $(2\delta + \tilde{p})$		+		0. 0,2	
+	3625	fin. $(2\tilde{\delta} - \tilde{p})$		+		1.14,8	
+	9	fin. $(2\delta + \epsilon)$		+		0. 0,2	
—	30	fin. $(2\delta - \epsilon)$		—		0. 0,6	
+	124	fin. $(2\omega - 2\delta + \tilde{p})$		+		0. 2,5	
—	76	fin. $(2\omega - 2\delta - \tilde{p})$		—		0. 1,5	
+	3	fin. $(2\omega + 2\delta - 2\tilde{p})$		+		0. 0,1	
—	73	fin. $(2\omega - 2\delta - 2\tilde{p})$		—		0. 1,5	
+	619	fin. $(2\omega - 2\delta + \epsilon)$		+		0.12,8	
—	463	fin. $(2\omega - 2\delta - \epsilon)$		—		0. 9,6	
+	1795	fin. $(2\tilde{p} - 2\delta)$		+		0.37,0	

Correctio hujus formulæ per observationes.

§ 50. Postquam hanc formulam cum observationibus pluribus contuliffem, facile adparuit, eccentricitatem orbitæ lunaris, quam $= 0,05454$ in hoc calculo supposueram (§ 40.) augendam esse parte sua $\frac{1}{300}$, atque adeo loco termini hujus formulæ primi $-6^{\circ}.16'.56''$ fin. $\tilde{p}$ hunc esse substituendum $-6^{\circ}.18'.11''$ fin. $\tilde{p}$. Deinde etiam terminus $-2'.38',6$ fin. $\tilde{\omega}$, qui maximus est eorum qui à parallaxi Solis pendent, nimis magnus est deprehensus, quippe quem observationes majorem non admittunt, quam $-1'.55''$ fin. $\tilde{\omega}$. Præterea in eadem hac formula plures termini occurrunt, quos theoria, licet summo studio tractata, accurate præbere non potest, ob rationes nulli non cognitatas, qui in hac re vires suas ac patientiam exercuit. Hos igitur terminos dubios malui ex observationibus definire,

finire, quam illorum gratia calculum tædiosissimum, à me jamdudum longe accuratius institutum, quam à quoque hactenus factum, ulterius adhuc persequi, novisque ac forte insuperabilibus augere difficultatibus. Denique etiam eos terminos, quos theoria satis manifeste ostendit, per observationes ita correxi, ut paucis secundis adjectis vel demtis cum cœlo magis consentirent. Hæc ideo præcipue moneo, ne quis putet, calculum hactenus expositum seu formulam § præced. inde derivatam accuratiorē esse magisve conformem observationibus, quam tabulas meas lunares correctas. Ut enim iste calculus Soli theoriæ innititur, plurimisque undique premitur difficultatibus; ita tabulæ utrique tam theoriæ quam observationibus inædificatæ sunt, atque adeo non possunt non esse præferendæ. Interim tamen ut pateat, quam parum tabulæ à calculo hoc in plerisque inæqualitatibus differant, subjungam hanc earum comparationem.

Inæqualitates longitudinis Lunæ.

Argumenta inæqualitatum juxta ordinem tabularum mearum lunarium.	Ex calculo theoretico, secundum correctiones initio hujus §, indicatas.	Secundum tabulas meas lunares, earumque ultimam correctionem.
I. {	+ 0. 11. 39	+ 0. 11. 14 fin. ϵ
	— 0. 0. 10	— 0. 0. 4 fin. 2ϵ
II.	— 0. 0. 58	— 0. 0. 56 fin. $(2\omega + \epsilon)$
III.	— 0. 1. 13	— 0. 1. 6 fin. $(2\omega - \epsilon)$
IV.	+ 0. 0. 55	+ 0. 0. 49 fin. $(2\omega + \dot{p})$
V. {	— 1. 20. 8	— 1. 20. 36 fin. $(2\omega - \dot{p})$
	+ 0. 0. 38	+ 0. 0. 26 fin. $(4\omega - 2\dot{p})$
VI.	+ 0. 2. 11	+ 0. 2. 0 fin. $(2\omega - \dot{p} + \epsilon)$
VII.	+ 0. 0. 44	+ 0. 0. 47 fin. $(2\omega - \dot{p} - \epsilon)$
VIII.	+ 0. 0. 40	+ 0. 0. 28 fin. $(\dot{p} - \epsilon)$
IX.	+ 0. 1. 26	+ 0. 0. 51 fin. $(2\omega - 2\delta)$
X. {	+ 0. 0. 8	+ 0. 0. 16 fin. $(\omega - \dot{p})$
	— 0. 1. 2	— 0. 1. 0 fin. $(2\omega - 2\dot{p})$
XI.	0	+ 0. 0. 4 fin. longit. δ
XII. {	— 6. 18. 11	— 6. 18. 11 fin. $\ddot{p}$
	+ 0. 13. 2	+ 0. 12. 52 fin. $2\ddot{p}$
	— 0. 0. 36	— 0. 0. 37 fin. $3\ddot{p}$
XIII. {	— 0. 1. 55	— 0. 1. 55 fin. $\tilde{\omega}$
	+ 0. 35. 42	+ 0. 35. 47 fin. $2\tilde{\omega}$
	+ 0. 0. 1	+ 0. 0. 2 fin. $3\tilde{\omega}$
	+ 0. 0. 5	+ 0. 0. 14 fin. $4\tilde{\omega}$
XIV.	+ 0. 1. 15	+ 0. 1. 26 fin. $(2\delta - \ddot{p})$
XV.	— 0. 6. 51	+ 0. 6. 51 fin. 2δ

Termini,

Termini, quos præter hos formula § 49. exhibita complectitur, partim in se minimi sunt, partim vero à theoria dubii relinquuntur. Observationum autem diligentissimo examine instituto satis evidenter patuit, eosdem ex tabulis prorsus esse rejiciendos.

Parallaxis Solis accuratius definita.

§ 51. Præter veriore motuum lunarium cognitionem, quam mihi præbuit comparatio hujus calculi cum observationibus præcipue stellarum à Luna occultatarum, accuratiorem quoque indidem obtinui Solis parallaxin; atque id quidem ex termino — $2'.38'',6$ fin. ω , cujus coëfficiens $2'.38''.6$ per theoriam definitus supponit parallaxin Solis $10'',8$, (vide § 37.) huicque constanter est proportionalis. Quodsi enim observationes eundem valorem hujus coëfficientis præbuissent, id simul indicio fuisset, parallaxin Solis esse $10'',8$. At vero, quoniam istæ inæqualitatem hoc termino comprehensam ad $1'.55''$ depresserunt, quemadmodum superius (§ 50.) dictum, propterea parallaxis Solis major esse nequit, quam $\frac{10'',8 \times 1'.55''}{2'.38'',6}$ five $7'',8$, hancque quantitatem à veritate ultra partem suam vigesimam quartam differre non posse inde colligo, quod de inæqualitate motus Lunæ, unde hæc parallaxis deducta est, intra $5''$ certus sim, quæ sunt totius, sc. $1'.55''$, pars circiter vigesima quarta.

Determinatio inæqualitatum in latitudine Lunæ.

§ 52. Ex æquationibus principalibus, § 18. exhibitis resolvenda adhuc est tertia, complectens latitudinem Lunæ ejusque inæqualitates. Multo autem facilius hæc res succedet, ac resolutio duarum priorum, idque ideo, quod termini in hac æquatione contenti minimi sint, ac præterea jam cogniti habeantur numerisque expressi valores literarum $x, y, \omega, P, n, e, \pi$. Modus resolvendi, quoad summa ejus capita,

O

nam

nam singula minutatim exponere non vacat, neque ad meum institutum necessarium videtur, hic est :

§ 53. Ponatur pro tang. l hæc series indeterminata

$$\text{tang. } l = I \sin. ip + K \sin. xp + L \sin. \lambda p + \dots$$

eritque posito dp constanti

$$\frac{dd \text{ tang. } l}{dp^2} = -I^2 \sin. ip - Kx^2 \sin. xp - L\lambda^2 \sin. \lambda p - \dots$$

et substitutis his valoribus in æquatione III. § 18.

$$0 = \left\{ \begin{array}{l} -(i^2 - 1) I \sin. ip \\ -(x^2 - 1) K \sin. xp \\ -(\lambda^2 - 1) L \sin. \lambda p \\ \&c. \end{array} \right\} + \left\{ \begin{array}{l} I \sin. ip \\ K \sin. xp \\ L \sin. \lambda p \\ \&c. \end{array} \right\} \left(\frac{3\pi^2}{2e^2 x^4 y^3} + \frac{3\pi^2 \cos. 2\omega}{2e^2 x^4 y^3} + \frac{3\pi^2 \cos. p}{8e^2 x^5 y^4} + \frac{15\pi^2 \cos. 3\omega}{8e^2 x^5 y^4} - \frac{2P}{e} + \frac{P^2}{e^2} \right)$$

§ 54. Hinc facile, substitutis literarum $n, e, x, y, \omega, \pi, P$, valoribus numericis, et posito $I =$ tangenti inclinationis mediæ $= 0,09$, ita ut fit $ip =$ motui medio Lunæ à nodo ascendente, determinabuntur valores quantitatum incognitarum $i, K, L, \&c.$ quo calculo absoluto erit

$$i = \sqrt{(1,0080587)} = 1,0040213,$$

hincque motus nodorum retrogradus $= 0,0040188$ posito motu medio longitudinis $= 1$; diurnus igitur nodorum motus medius juxta theoriam foret $= 3'. 10''. 45'''$, qui secundum observationes est $= 3'. 10''. 46'''$, consentiente theoria cum experientia ad amissim.

§ 55. Porro eodem calculo invenietur

$$\text{tang. } l = + 0,9000000 \sin. ip$$

$$+ 27284 \sin. (2r - i) p$$

$$+ 1090 \sin. (2r + i) p$$

$$- 1103 \sin. (n + i) p$$

$$- 1174 \sin. (n - i) p$$

$$+ 348 \sin. (\alpha + i) p$$

$$+ 1023 \sin. (\alpha - i) p$$

$$\begin{aligned}
& + 47 \text{ fin. } (2r + \alpha + i) p \\
& - 256 \text{ fin. } (2r + \alpha - i) p \\
& + 653 \text{ fin. } (2r - \alpha + i) p \\
& + 1606 \text{ fin. } (2r - \alpha - i) p \\
& + 9 \text{ fin. } (2r + n + i) p \\
& + 553 \text{ fin. } (2r + n - i) p \\
& - 75 \text{ fin. } (2r - n + i) p \\
& - 1249 \text{ fin. } (2r - n - i) p \\
& + 2014 \text{ fin. } (2r - 2\alpha + i) p \\
& + 1739 \text{ fin. } (2r - 2\alpha - i) p \\
& + 9 \text{ fin. } (r + i) p \\
& + 26 \text{ fin. } (r - i) p \\
& + 20 \text{ fin. } (2n + i) p \\
& + 23 \text{ fin. } (2n - i) p \\
& + 3 \text{ fin. } (2\alpha + i) p \\
& + 1415 \text{ fin. } (2\alpha - i) p \\
& + 99 \text{ fin. } (4r - 2\alpha - i) p \\
& - 8 \text{ fin. } (\alpha + n + i) p \\
& - 21 \text{ fin. } (\alpha + n - i) p \\
& + 15 \text{ fin. } (\alpha - n + i) p \\
& - 36 \text{ fin. } (\alpha - n - i) p \\
& + 53 \text{ fin. } (4r - i) p \\
& - 68 \text{ fin. } (4r - \alpha - i) p \\
& + 466 \text{ fin. } (2r - 3i) p \\
& - 50 \text{ fin. } (2r - 2n - i) p \\
& + 163 \text{ fin. } (r - \alpha + i) p \\
& + 153 \text{ fin. } (r - \alpha - i) p.
\end{aligned}$$

Etsi in hac quoque serie complures termini comprehendantur, qui ob parvitatem penitus obmitti potuissent, tamen eos adposui ad demonstrandum, quanta cura in hoc negotio sim versatus.

§ 56. Ut hæc formula ad calculum tabularem aptior reddatur, ponatur ut supra § 49.

Distantia loci Lunæ veri in orbita à nodo ascendente correcto $= \delta$

Anomalia media Lunæ $= \dot{p}$

Distantia loci Lunæ veri in orbita à loco Solis vero . . . $= \tilde{\omega}$

Anomalia media Solis $= s$

critque adplicata loco medio nodi ascendentis hac æquatione,

$$+ 9' . 13'' \text{ fin. } s - 5'' . 5 \text{ fin. } 2s,$$

ut locus ejus correctus habeatur, et resolutis coefficientibus inæqualitatum singularum ad partes graduum circuli,

l , five latitudo vera Lunæ

$$\begin{aligned} &= + 5 . 8 . 26,1 \text{ fin. } \delta \\ &\quad - 6,6 \text{ fin. } 3 \delta \\ &\quad + 8 . 47,6 \text{ fin. } (2 \tilde{\omega} - \delta) \\ &\quad - 7,9 \text{ fin. } (2 \tilde{\omega} - \delta + s) \\ &\quad - 4,7 \text{ fin. } (2 \tilde{\omega} - \delta - s) \\ &\quad - 2,9 \text{ fin. } (2 \tilde{\omega} - \delta + \dot{p}) \\ &\quad + 16,5 \text{ fin. } (2 \tilde{\omega} - \delta - \dot{p}) \\ &\quad - 16,2 \text{ fin. } (\delta - \dot{p}) \\ &\quad - 29,1 \text{ fin. } (\delta - 2 \dot{p}) \\ &\quad + 3,2 \text{ fin. } (\delta - 3 \dot{p}) \\ &\quad - 2,1 \text{ fin. } (2 \tilde{\omega} - 3 \delta) \\ &\quad + 1,1 \text{ fin. } (\delta + \dot{p}) \\ &\quad - 2,0 \text{ fin. } (2 \tilde{\omega} - \delta - 2 \dot{p}) \\ &\quad + 1,1 \text{ fin. } (4 \tilde{\omega} - \delta - 2 \dot{p}) \\ &\quad + 6,5 \text{ fin. } (2 \tilde{\omega} + \delta - 2 \dot{p}) \\ &\quad - 1,3 \text{ fin. } (2 \tilde{\omega} - 3 \delta + \dot{p}) \\ &\quad + 1,3 \text{ fin. } (2 \tilde{\omega} - 3 \delta - \dot{p}). \end{aligned}$$

Correctio

Correctio hujus formulæ per observationes.

§ 57. Supponit hæc formula valorem literæ I. five tangentem inclinationis mediæ orbitæ Lunæ ad eclipticam = 0,09 (vide 54.) atque inde præcipue ortus est coëfficiens termini primi $5^{\circ}.8'.26'',1$; quem postquam cum observationibus comparavissem, adeo veritati esse consentaneum deprehendi, ut nihil fere in eo mutandum videbatur. Addidi tamen ipsi $4''$, quæ omnem correctionem constituunt, quam observationes fuadebant; atque ita tabulas pro invenienda latitudine Lunæ vera ad normam hujus formulæ construxi:

Latitudo vera Lunæ =

I.	{	[°] + 5. 8. 30 fin. δ
		['] — 0. 6 $\frac{1}{2}$ fin. 3δ
II.	+	8. 47 fin. $(2\tilde{\omega} - \delta)$
III.	—	0. 8 fin. $(2\tilde{\omega} - \delta + s)$
IV.	—	0. 5 fin. $(2\tilde{\omega} - \delta - s)$
V.	—	0. 3 fin. $(2\tilde{\omega} - \delta + p)$
VI.	+	0. 16 fin. $(2\tilde{\omega} - \delta - p)$
VII.	—	0. 16 fin. $(\delta - p)$
VIII.	—	0. 29 fin. $-(\delta - 2p)$
IX.	+	0. 3 fin. $(\delta - 3p)$
X.	+	0. 6 fin. $(2\tilde{\omega} - 2p + \delta)$

reliquas inæqualitates, quas continet formula § 56. penitus neglexi, quandoquidem maxima earum secunda duo vix excedit.

§ 58. Supereffet demonstranda adhuc veritas tabularum, ex quibus parallaxis Lunæ depromitur. Sed cum res sit admodum facilis ope seriei supra (§ 45.) pro valore ipsius x exhibitæ, nolui ejusmodi minutiarum expositione, in qua forte jam modum transgressus sum, diutius lectorum patientiam defatigare.

F I N I S.



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